

# Plan an MTC session!

## *Brownies*

You make some delicious brownie batter, pour it to fill a rectangular pan, and then bake it. The brownies are cooling on the counter when a mischievous kid walks in and cuts out a rectangular brownie somewhere in the pan and eats it. Later, you want to fairly divide the remaining brownie between two people, using only a single straight cut. Is this possible?

Today we brainstormed about what makes a good session. Tonight or tomorrow at breakfast, if you like, you can have fun playing with this problem. Start generating some ideas about a “hook” that will get people involved in the first few minutes of your session, as well as a goal that you want to be sure everyone understands by the end of your session. Think of some other possible directions you could go with the ideas raised by this problem, extensions of this problem, or additional problems that this problem might lead to.

Tomorrow we will work on planning a math teachers' circle session based around this problem. You'll have about an hour to work out your ideas, and then about 10 minutes to share your plans, again focusing on those first few minutes with a bit of an outline of some key results to encounter and some possible extensions depending on where the participants go with your problem.

## **Brownies: Answer/Hint/Solution.**

**Think about the problem for a while first before you read this!**

Any line through the center of a rectangle divides it into two regions of equal area. In fact, they are two congruent regions! Thus the line through the center of both rectangles will divide the brownies into two equal-area pieces.

Now, what if ...

# Planning a Math Teachers' Circle Session

by the MTC team

## Introduction

The most important topic we haven't covered so far in this workshop falls into the gap between our morning math sessions and our afternoon planning sessions. We need to learn how to plan a math teachers' circle session!

Usually the planning begins with a good problem. Why did we choose the problems you've seen in the mornings so far? We've picked a few new sample problems here for you, and shortened them a bit given your limited time to work. Why did we pick those? I hope our brainstorming has helped answer this question for you.

As you're planning, you'll want to find some hints, or steps along the way, or stopping points where you might interrupt to point out some interesting strategy or a connection to a different kind of problem. You'll think about how to structure the session: how will you introduce the problem? How will you get people engaged? At what moments do you want to have longer pauses for people to think? What are good possible stopping points? This leads to a plan.

But the key element of the plan is that you can abandon it! For a two-hour session, you should have prepared at least 4 hours of material, if not twice that. You'll have lots of different directions you can go with it. The purpose of all the preparation is that you can be flexible, because you anticipate directions people might go, and you've made some decisions ahead of time about which ones you'll be prepared to follow. Depending on your group, you'll make use of different parts of that preparation. Don't try to cover too much! Just because you prepared it doesn't mean they need to see it.

Usually we work with a problem with at least three or four different groups of people before we discover all the directions it might go. These sessions definitely get better with time! Planning with a partner can be tough, but it also helps you uncover a lot of those possibilities before your first encounter with your circle's participants and makes a big difference in the effectiveness of your preparation.

## Directions

Now it's your turn: time to plan a session! Of course, in real life this takes a lot more than the amount of time you have right now. And you can choose a problem that has some particular interest to you or to your audience (or, ideally, both!). Here, to save a bit of time, we've already chosen a problem to use as a starting point.

In your planning, please let us know what you would do with this problem idea. How would you introduce it? What kind of thinking would you be looking for? What connections would you make? What related problems might you bring in? What deeper ideas could you aim at?

You've had chances to comment on what you observed about how to teach, and what you'd do differently from the session leaders you've seen. We've brainstormed on what makes a good problem and a good session. Now you have a chance to put that into action! After some planning time, you'll have 10 minutes to share your ideas. During most of that time, please discuss or demonstrate what you'd do to introduce the problem in the first 5-10 minutes of your session. Then, tell us briefly about one key idea or result that you would want participants to take away as an important landmark on their exploration of this problem, and what related problems you might use to make this into a full session.

## **A sketch of one possible approach to the brownie problem**

**Joshua Zucker**

I might start by asking

Does there exist a straight line to cut a full pan of brownies into two equal pieces?

After getting the obvious “yes” answer, I'd ask how, and hope to see horizontal, vertical, and diagonal answers at least. Then I would ask if there are any other solutions, and wait as long as necessary for everyone to eventually suggest and then prove that any line through the center of the rectangle works.

I might introduce various other shapes of regions here, like circles, triangles, and irregularly shaped things, to lead toward the idea that any direction the knife points, you can slide it across and find a spot somewhere in the middle that makes two equal-area but not necessarily congruent regions. We could also explore the question of which shapes have a “center” like the rectangle, where all these lines meet no matter what direction the knife points. Or, I might skip this part or save it in case the participants point us toward these ideas.

Next, I'd ask the question given on the handout, about a rectangle with a rectangular hole, and give them a lot of time to figure out that they can reuse the idea from the single rectangle to solve this problem as well. I'd put several additional problems on a handout, so that participants who solve this very quickly will have other things to work on instead of waiting around bored. Additional problems might include questions like

Some people prefer the crispier outside edge of the brownies, and other people prefer the chewy center. Is it possible to use a single straight cut to split the rectangle with a hole in it in such a way that you divide the outer edge evenly and also the interior evenly?

What if you have an equilateral triangular pan? Perhaps with an equilateral triangular hole?

If you have an arbitrary shape of triangular pan and want to split both the edge and the interior evenly, does there exist a single straight cut that will do this?