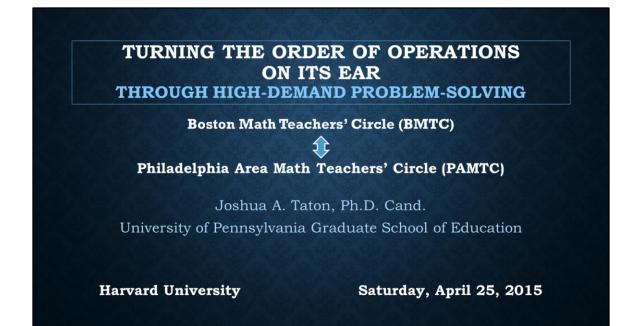


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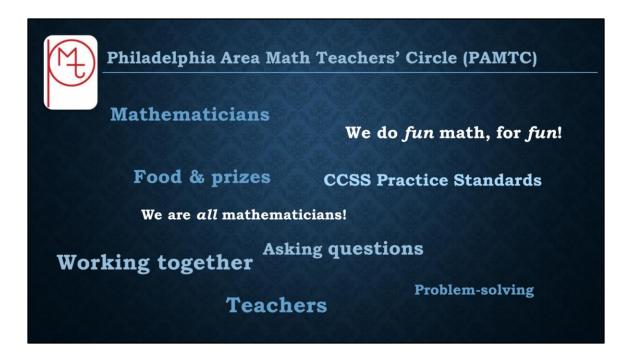
Josh Taton, PhD Candidate UPenn GSE (jtaton@upenn.edu) Amy Myers, PhD, Bryn Mawr College Michael Nakamaye, PhD, University of New Mexico (for the 1,2,3,4 problem) the Philadelphia Area Math Teachers' Circle (PAMTC, philamtc@gmail.com) and the American Institutes of Mathematics



Hi! My name is Josh Taton, and I'm working on my dissertation for my PhD at the School of Education at Penn. My work involves studying how teachers use curriculum materials—the types of decisions that they make with materials. I've also taught in the Teacher Education Program at Penn.

Before coming to Penn I was a middle and high school math teacher for about 7 years. And now that I'm at Penn, I'm constantly in classrooms, doing research, learning from teachers, working with students and teachers.

I'm also one of the co-founders and leaders of the Philadelphia Area Math Teachers' Circle. We started holding our monthly sessions in 2011 and we are continuing to grow. I'm very excited to learn with and from you, today, because I think each MTC has its own approach but common interests; if you have any questions about how we run our MTC, I'm happy to answer, and I'd like to learn more about what you do here, as well.



Today, I wanted to share with you a problem we've done with teachers in our MTC and our overall approach. First, I know that you are aware of what Math Teachers' Circles do but I thought I would share our quick recruiting pitch with you.

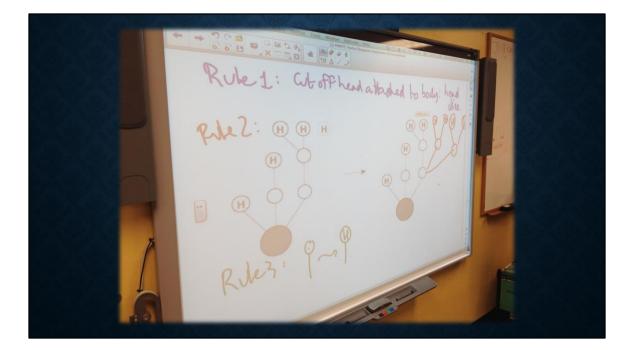
[Go through these, briefly.]

Who are we? We are made up of mathematicians and teachers [although I'm neither!] ...But we say we are ALL mathematicians What do we do? We do fun math, for fun! How? By working together ...and by doing authentic problem-solving ...that involves us asking and answering our own questions Why? CCSS Practice Standards (and to support students in problem solving and enjoying math)

Why else? Food and prizes



Here are some images from a recent MTC session.



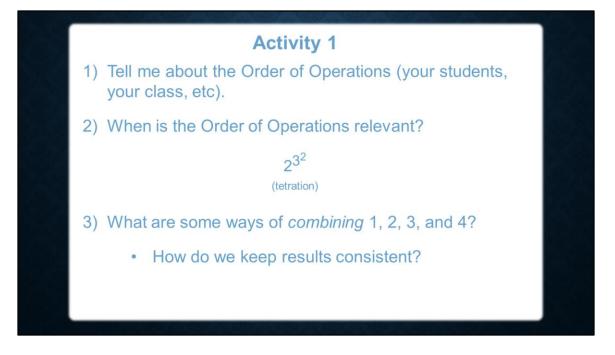


Standards for Mathematical Practice (CCSS)

- 1. Make sense of problems & persevere.
- 2. Reason abstractly & quantitatively.
- 3. Construct and critique arguments.
- 4. Model.
- 5. Use tools appropriately & strategically.
- 6. Attend to precision.
- 7. Look for & make use of structure.
- 8. Look for & express regularity.



I like to show the Math Practice Standards at our workshops, because I think they really embody what we try to work on together. This is also an area that I've heard from teachers needs more support. [Show of hands...how many people have seen the Math Practice Standards? How many have had PD on them? How many want more PD on them?]



Of course, as you know, we really try to put ourselves in the position of our students; to see a new problem for the first time, to feel confused, and to figure out ways to work through that confusion together. We believe that as we strengthen our own mathematical problem solving skills, we can pass along some of these strategies and provide similar opportunities for students.

I want to warm-up, though, by having a quick discussion about the Order of Operations. Tell me what you do with it. How you teach it. Any frustrations you or your students have. Go.

[Discussion.]

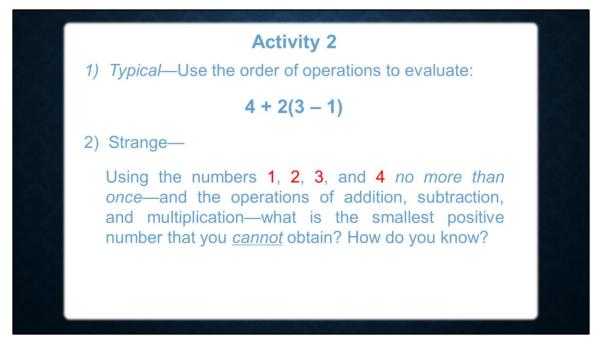
Now I have a question for you...when does the Order of Operations become relevant? With what sorts of problems?

First, problems involving 3 or more numbers (let's stick to whole numbers). Second, two or more (different) operations. Are you sure you need two or more operations? Here's a fun one to discuss:

2^3^2

OK, next question: what are some ways we can combine the numbers 1, 2, 3, and 4?

As I'm writing these down, how do I make sure that we have consistency in our results? (How do we communicate the Order of Operations, so that our results are consistent? [Looking for parentheses, here.]



Now we are going to do some problem-solving. First, as we've discussed, I want to show a fairly typical Order of Operations problem. When we were students, we might all have done pages and pages of worksheets with problems like this one, right? As a teacher, I certainly know I assigned homework of that sort!

Next, here's how I'm going to turn that problem on its ear; at the end, you can tell me if you think it's still an order of operations problem, or if it could be made an order of operations problem:

[Read problem and ask if there are any questions about the problem.]



Have some groups work on extension problems, if needed. See if they can formulate questions of their own.

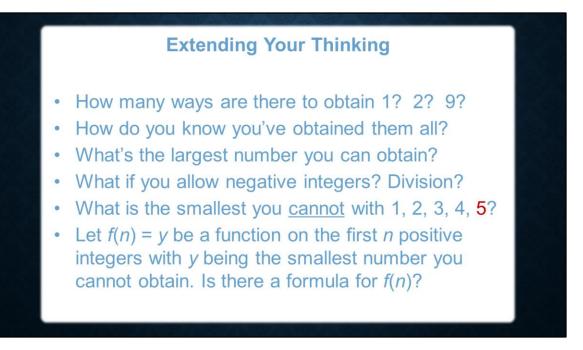


[After problem-solving...and discussing some solutions....]

Did anyone think of other questions that they could ask about this situation?

[Solicit and discuss other questions.]

I think that these sorts of extension questions are great examples of ways you can differentiate instruction in your classroom. Some students will be able to reach for these sorts of questions and some will not, but they all have the opportunity to engage in high-level thinking about math.



Here are some of the questions that I was thinking about, when I was working on this problem.

[If there is extra time, let's choose one of these to work on.]

Strategies Used in Problem-Solving?

- 1. Make an organized list of *results*
- 2. Make an organized list of ways to combine numbers
- 3. Look for patterns
- 4. Ask questions
- 5. Decompose larger numbers
- 6. Work backwards

I also wanted to ask...what sorts of strategies did you use in working on this problem? We like to talk about these at our sessions.

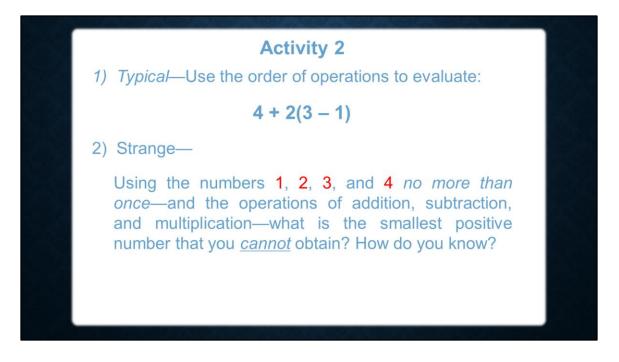
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- 6. Attend to precision.
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- 8. Look for & express regularity.



Here, again, I'm showing the Math Practice Standards. Does anyone have any observations or comments about the Standards in relationship to the activity? I guess what I'm asking is, "Do you think that this problem is worthwhile? Could you use it?"

[Solicit feedback. Plan to talk about a few important ways that these emerged as the groups worked together.]

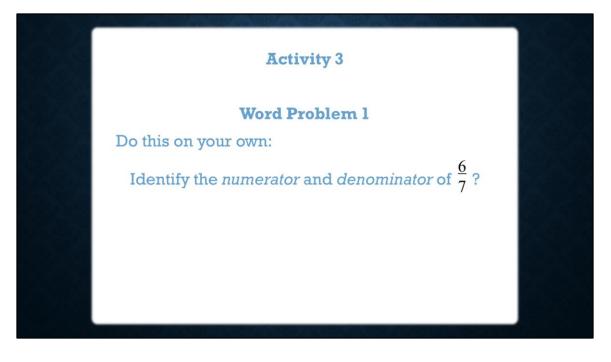


I just want to show these two problems again. I have two questions:

- Is this an Order of Operations problem? Or could we adjust it to have it be an Order of Operations problem? [I think it can be, if you insist that students document their lists of non-answers in accordance with Order of Operations...or if you have a discussion about why two different people could obtain two different ways of combining values, unless we had agreement.]
- 2. From comparing these and looking at the way these problems are written, do you have any thoughts on how to turn "ordinary/typical" problems into "strange" ones?

[Solicit ideas.]

A natural question that I get a lot is "How can I do this sort of problem-solving session in my classroom?" Note that this is a different sort of question than "Why should I do this in my classroom?" I think you have to sort of buy into the idea that doing these activities is worthwhile and can build students understanding before trying to tackle how.



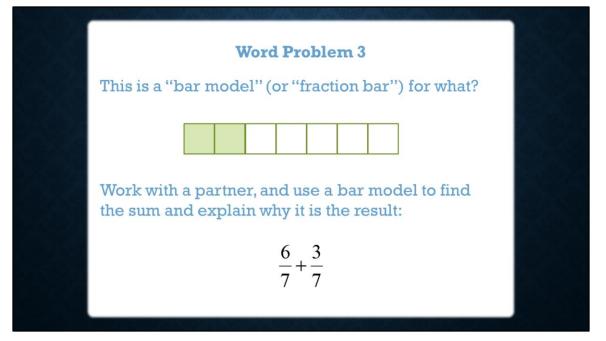
To tackle this question, I want to answer a question with a question—by doing another short activity. I'm less concerned about the specific answers, but rather that you think about and make observations about the types of tasks that I'm giving you.

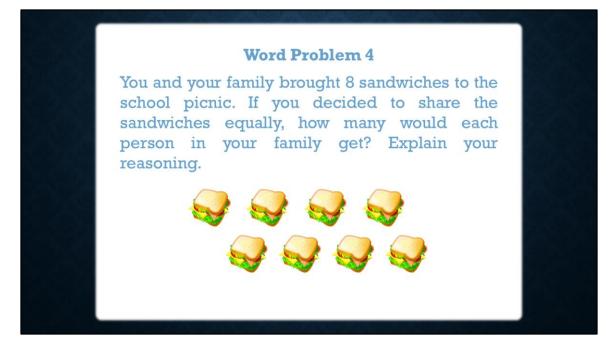
I'm going to give you four problems—in sequence—with very specific instructions for each task. [Read each problem on the next several slides.]



Do this on your own:

$$\frac{6}{7} + \frac{3}{7}$$





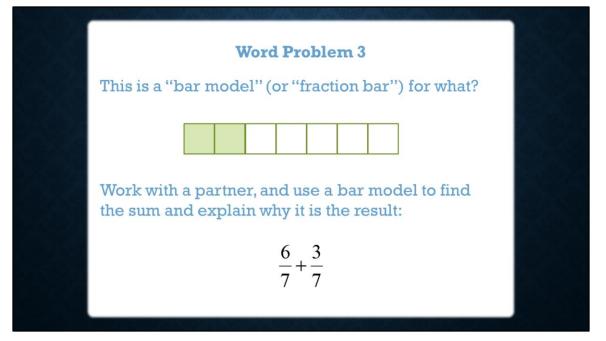
What do you notice about these problems? What is the same? Different?

Would you say that any of the problems is more complex than any of the others? Why?



What was similar about these problems? Different?

Here's the main take-away. I'm going to show the last two problems again.



Word Problem 4

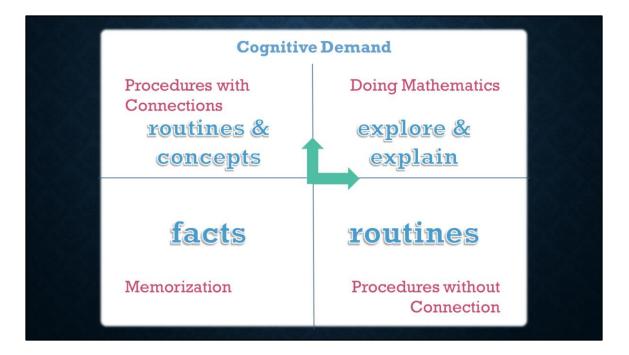
You and your family brought 8 sandwiches to the school picnic. If you decided to share the sandwiches equally, how many would each person in your family get? Explain your reasoning.



Several features:

- 1) No clear answer
- 2) No clear solution path (approach)
- 3) Explain your reasoning

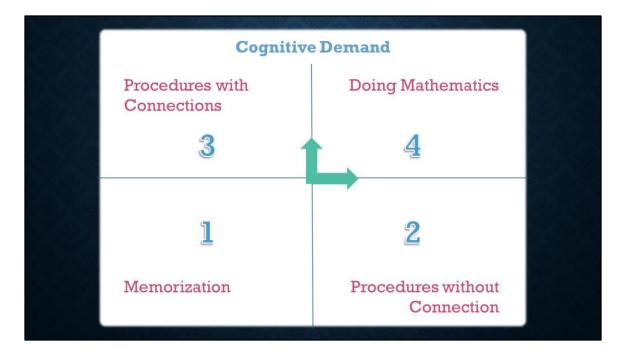
Research has shown, time and again, that when students have more consistent opportunities to explain their reasoning with these sorts of problems—not only do they do better in solving complex problems, but they also succeed in solving the skill-based (simpler) problems.



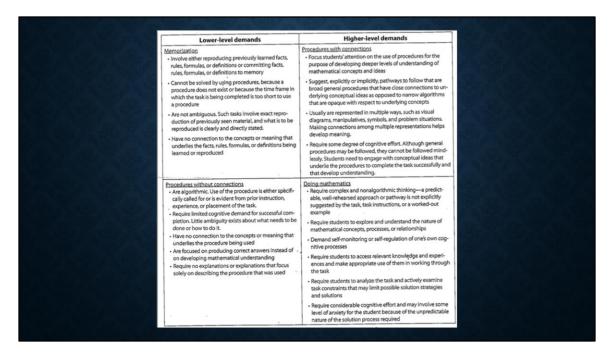
This slide shows four categories of what is called cognitive demand. It's one way to think about the rigor of a problem or task in a math classroom and here are some quick descriptions of these categories. [Go through them.] Notice the green arrows pointing to the right and upward, those show the pathway of increasing cognitive demand.

As I said earlier, research has shown that when students are more consistently given higher demand problems, their achievement improves. It also can be difficult to offer and maintain the cognitive demand of problems and activities in your classroom for a variety of reasons, and [if time] we will look at some tools to support this maintenance of cognitive demand, later on.

[Questions? The idea of memorization being important may come up here; I'm not saying it's not important, but that sometimes you can achieve memorization through work with things that are more intellectually challenging. For example, let's say you get a new washer machine. You don't read the manual and memorize all the terms/labels for the buttons, right? You work with the machine, test things out, problem-solve; over time, you find that you have actually memorized the placement of the buttons and what the different labels mean.]



The four problems that we looked at fit into these categories.



Here's a resource that I can send you, if you email me. [I know it's hard to see on this screen.] This table provides additional details about cognitive demand categories. Research has shown that: a) maintaining high demand improves student achievement, b) teachers in the US often make adaptations during lessons that lower the cognitive demand and thereby provide less of an opportunity for students to grapple with challenging rigorous ideas and thinking

[See, for additional details: Smith, M.S., & Stein, M.K. (1998). Selecting and creating mathematical tasks: From research to practice. Mathematics Teaching in the Middle School, 3(5), 344-350.]

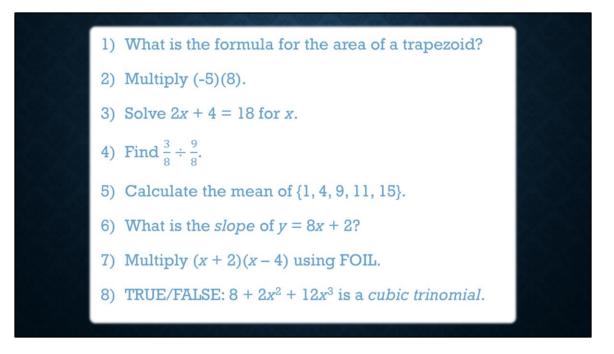
Examples

1) Solve the system of equations using *elimination:*

$$2x + 5y = 15$$

-6x - 15y = -4

2) A system of two linear equations has no solution. One of the equations in the system is 2x + 5y = 15. Give two examples of what the other equation in the system could be if the system has no solution. Give the equations in standard form. Explain how you found the equations.



Now I'd like you to try your hand at increasing cognitive demand. Choose one or two of these problems and work together to rewrite them. Discuss how you could change the problem and why the cognitive demand is increased.



Notes from discussion:

Helpful ways to address student difficulties with PEMDAS-

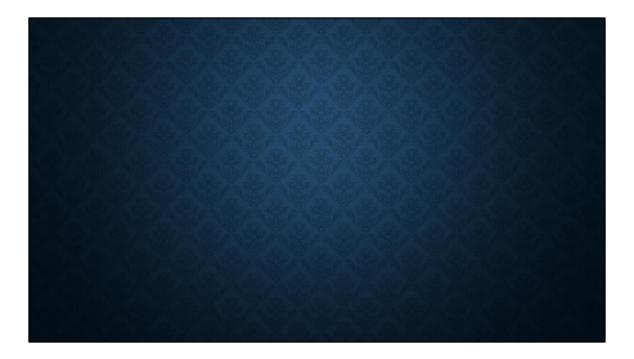
- 1) Put boxes around MD and AS
- 2) Change PEMDAS to GERMDAS (grouping, exponents, radicals, etc.)
- 3) Require students to write multiple steps in ONE line, instead of multiple lines
- 4) Ask students to figure out how to come to agreement, given that they can disagree on order of steps in a multi-step expression
- 5) Ask students to make sure any expression can "fit" within the rules

Some concerns expressed about how to increase rigor effectively, while also addressing standards/testing needs

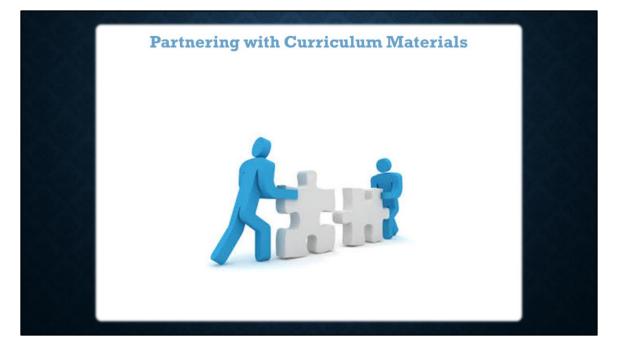
Some concerns expressed about putting abstract/rigorous questions ahead of skillbased questions (fear of letting kids grapple; not sure if they can do as much as researchers think they can, behavior concerns, etc.)



Questions?



Additional Activities (as needed and if interested)



There are two main areas in which we can work on supporting student understanding. One involves partnering with the curriculum. By this, I mean utilizing the supports that are offered in curriculum materials while also adapting them to suit students' needs and also to help develop conceptual understanding.

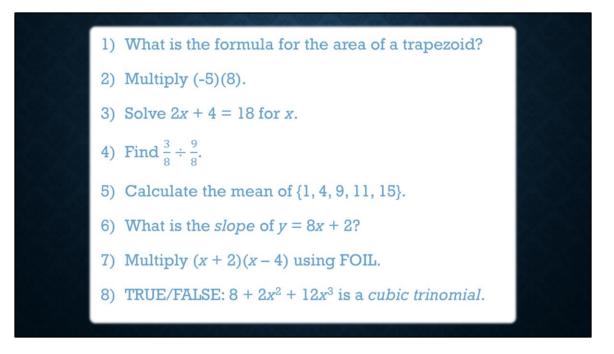
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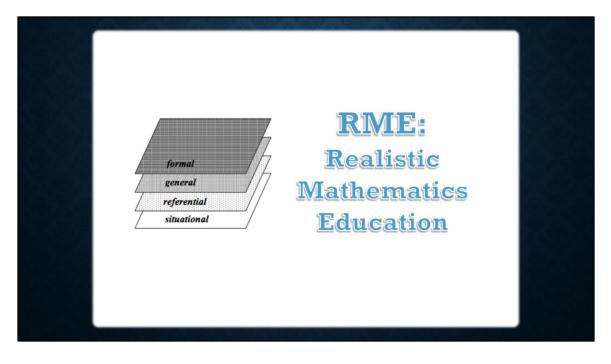
Now I'd like you to try your hand at increasing cognitive demand. Choose one or two of these problems and work together to rewrite them. Discuss how you could change the problem and why the cognitive demand is increased.

Examples

1) Find *x*:

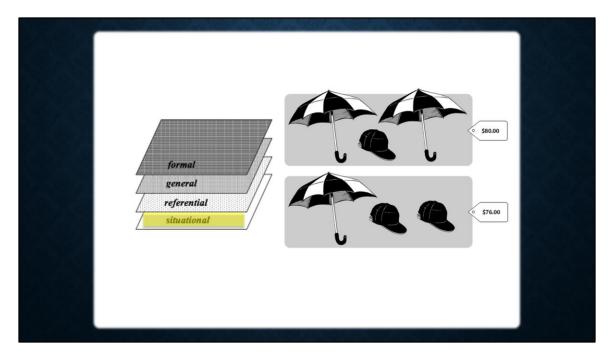
 $160(0.75)^x < 1$

2) Your baby brother accidentally ingests 160mg of aspirin at 8am, when you are babysitting. He is known to have a bad reaction to aspirin. The aspirin is metabolized at a rate of 25% each hour. Will there be any left in his system at 4pm—if 10mg in his system is enough to cause the reaction? Explain why or why not.

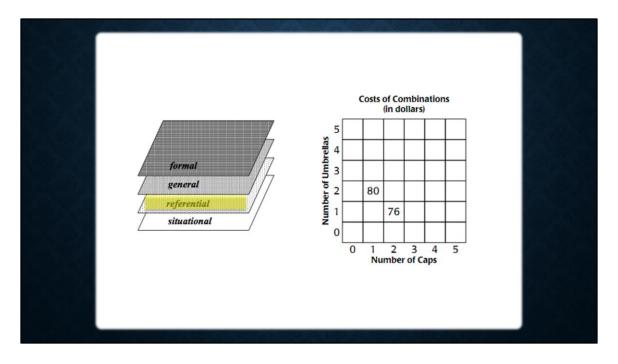


So increasing cognitive demand is Approach 1. Approach 2 is a theory called RME which stands for realistic mathematics education. I'm going to fly through this. Here there are also four levels of problems.

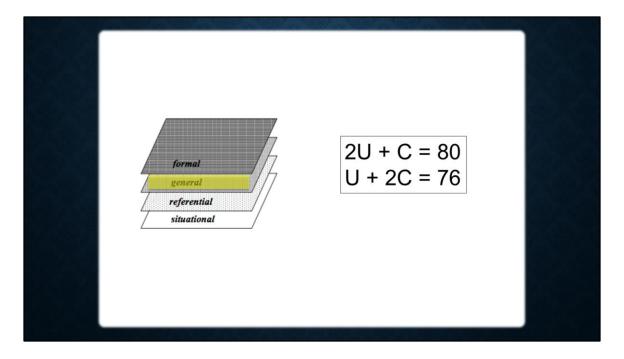
[Fruedenthal, 1970s-80s; ideas became much better known, more recently, 90s and 00s]



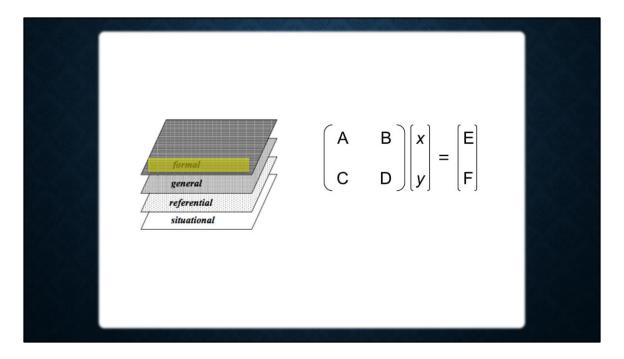
Here's a quick problem: How much is a hat? How much is an umbrella? In this problem, students are presented with a situation that involves identifiable objects. RME theory says that this is a situational problem. Very young students actually *can* solve this problem, just by using the pictures and talking through their reasoning.



Here, the same problem is presented, but students are taught and encouraged to use a tool for organizing the information. In this case, they are creating a table (this table hints at using a graph, too, because it evokes coordinates). The RME theory calls this referential, because the students are encouraged to reference umbrellas and caps, but they aren't making direct use of umbrellas or caps in the



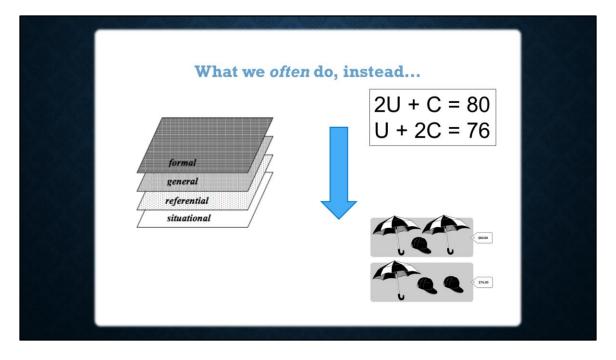
This is the same problem, presented in a very general format (familiar to us from algebra). We don't need to know that there are caps and umbrellas, but instead we just have variables U and C or (more commonly) x and y. There are a variety of ways to solve this sort of a problem, using algebra.



Finally, we have the formal model for this sort of problem, which is a way of looking at the problem that college mathematics students or mathematicians might use in an areas of math known as linear algebra or vector algebra or Diophantine equations.

What we should do	
situational	

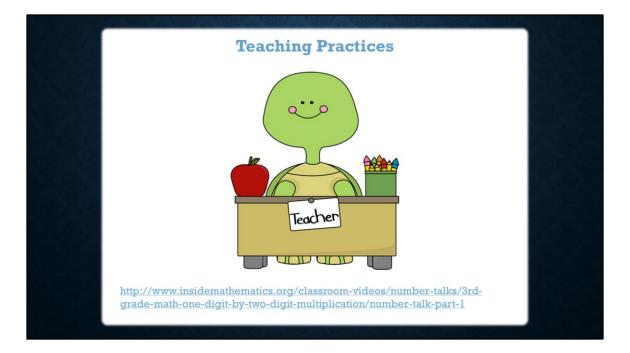
RME theory says that we *should* give students lots of exposure and practice at the lower levels, first, before moving to the upper levels.



More often, though, what we do is the exact opposite. Right? We teach students these complex methods, first, and then we put the word problems at the end of the chapter (and a lot of times, we don't even get to that section).

[Possible wrap-up question, if time permits:] What cog-demand level is The Brownie Problem; why? Is it an RME-type problem; how/why/why not?

[In case anyone asks, note that RME and cognitive demand are NOT incompatible. Sure, one says move from lower to higher (RME) and the other says to stay at the higher level (cog demand). BUT these theories are not describing the same features/characteristics of problems. In theory, you can certainly have high cognitivedemand problems that are situational and low cognitive-demand problems that are formal, so these are clearly distinct ideas. In fact, RME and cognitive-demand can be used in overlapping and mutually-supportive ways.]



The second main way that we can support students in developing understanding and this should not come as any surprise—is by taking a really reflective stance toward our teaching practices. Not being afraid to critically investigate the norms in our classroom, the tools that we use, and how we facilitate discussions.

I want to show a video of a third grade math lesson. I'm choosing third grade, because it allows us to really look at the teaching and not also have to think about the math at the same time. As you watch the video, feel free to write down observations about what you see. I'd strongly encourage you to be descriptive; don't analyze what you think about the teaching—whether you think it's good or bad or what have you—instead, try to just make observations that describe what you see happening in this class. What do you notice?

I should say that Insidemathematics.org is another great resource that has a lot of model lessons on video and other resources.



There is a substantial body of evidence in cognitive science, psychology, learning studies in math education that indicates that conceptual-oriented teaching before skill-oriented teaching supports learning better than the other way around.





There was a lot of research in the 1990's and 2000's comparing the typical curriculum in the US with those used in other higher-performing nations. And one criticism of the typical US curriculum is that math in the US is "a mile wide and an inch deep." [Ask what they know what this means?] Yes, a lot of topics are covered, but not with sufficient depth.

http://www.scoobyunion.com/spowell26/wpcontent/uploads/2010/12/shalllowWaterwalker.jpg

FIGURE 2 State Composite: Mathematics topics by at least two-thirds of 21 U.S. state		ded at	each	grade		pic P the U		nent			
	Note that topics are introduced and sustained in a way that produces no visible structure. TOPIC GRADE: 1 2 3 4 5 6 7 8										
			3	4			/	8			
Whole Number Meaning Whole Number Operations	-										
		-									
Measurement Units							-				
Common Fractions			-		-						
Equations & Formulas					-						
Data Representation & Analysis	-			-			-				
2-D Geometry: Basics			-			-					
Polygons & Circles						-					
Perimeter, Area & Volume					10	-					
Rounding & Significant Figures											
Estimating Computations			100		100	-	100				
Properties of Whole Number Operations											
Estimating Quantity & Size											
Decimal Fractions					100	-					
Relationship of Common & Decimal Fractions											
Properties of Common & Decimal Fractions											
Percentages						-					
Proportionality Concepts											
Proportionality Problems						-	10				
2-D Coordinate Geometry								-			
Geometry: Transformations	101	10	88	10	10	-	100				
Negative Numbers, Integers & Their Properties											
Number Theory					-						
Exponents, Roots & Radicals											
Exponents & Orders of Magnitude											
Measurement Estimation & Errors					10	-					
Constructions w/ Straightedge & Compass						_					
3-D Geometry						1					
Congruence & Similarity						-					
Rational Numbers & Their Properties							10				
Patterns, Relations & Functions	-				100	-		-			
Slope & Trigonometry											
Number of topics covered by at least 67%		1222									
of the states	14	15	18	18	20	25	23	22			
Number of additional topics intended by states to complete a typical curriculum at each grade level	8	8	7	8	8	5	6	6			

Let's take a look at these goals of the Common Core, one at a time. First, let's talk about Focus.

I apologize that this image is hard to read, but don't focus on the details. I just want you to get the overall impression of how this looks. What this chart is intending to show are the mathematics topics covered by a large number of US states from K-8. You can already get a sense of why the US curriculum is criticized for being a mile-wide and an inch-deep.

In sum, topics are taught repeatedly throughout the grades. More topics at each grade level means topics receive less depth. There is also an early introduction of many demanding topics (e.g., transformational geometry, measurement error, functions). The empty rows reflect a lack of consensus on when to teach a topic, such as properties of common and decimal fractions. The approach seems to be a laundry-list, rather than a thoughtful progression; in addition, this coverage of topics does not reflect the complex structure of mathematics, which requires pre-requisite knowledge before moving to more sophisticated knowledge.

	State of the second sec		Gr	ade	lopic	Placement in
ocus	Topic	1 2	3 4	5	The C	Common Core
rence	Whole Number Meaning	• •	• •	•		
tanding	Whole Number Operations	• •	• •	•		Contract of the street
stallullig	Properties of Whole Numbers Operations	• •	• •	•	•	
	Fractions	• •	• •	•	•	1949 1949 1949
	Measurement Units	• •	• •	•	• •	
	Polygons & Circles		• •	•	• •	•
	Data Representation & Analysis	• •		•	• •	•
	3-D Geometry	• •		•	• •	•
	Measurement Estimation & Errors	•	•	1		Contraction of the local sector
	Number Theory	•	•	•	•	Section and the
	2-D Geometry Basics	•	•	•		•
	Rounding & Significant Figures		• •	•		
	Relation of Decimals & Fractions		• •	•	•	
	Estimating Computations			•	•	•
	Perimeter, Area & Volume			•		•
	Equations & Formulas			•	• •	
	Decimals		•		0	
	Patterns, Relations & Functions		•			•
	Geometric Transformations		•		• •	•
	Properties of Decimals & Fractions			•	•	
	Orders of Magnitude			•		•
	2-D Coordinate Geometry			•		•
	Exponents, Roots & Radicals			•		•
	Percentages					
	Negative Numbers, Integers & Their Properties					
	Proportionality Concepts					•
	Proportionality Problems					· Contraction of the second
	Rational Numbers & Their Properties					
	Constructions Using Straightedge & Compass			+		
	Systematic Counting			-		
	Uncertainty & Probability			1		
	Real Numbers & Their Properties					
	Congruence & Similarity					
	Slope			1	1	
	Validation & Justification			1		
	validation of Justification			1	1	A CONTRACTOR OF A CONTRACTOR OFTA CONTRACTOR O

And, now, here is the recommended coverage of topics in the Common Core. Thoughts? Observations? [Show these three slides again, if needed.]

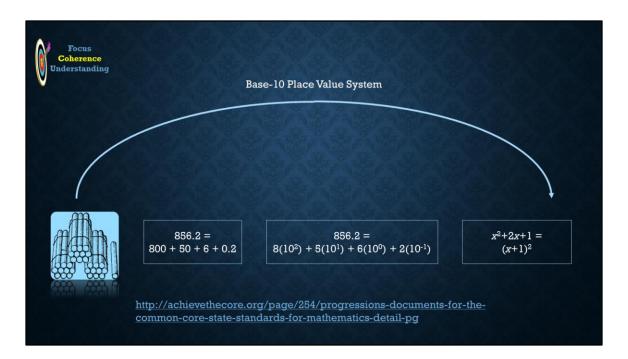
FIGURE 1 A+ Composite: Mathematics topics intended at each grade by at least two-thirds of A+ countries.							Topic Placement in Top Achieving Countries				
Note that topics are introduced and sustained in a					0						
TOPIC GRAD		2	3	4	5	6	7	8			
Whole Number Meaning	-										
Whole Number Operations				-							
Measurement Units			-		-						
Common Fractions											
Equations & Formulas								-			
Data Representation & Analysis						100					
2-D Geometry: Basics											
Polygons & Circles											
Perimeter, Area & Volume					-	10					
Rounding & Significant Figures				10							
Estimating Computations				88							
Properties of Whole Number Operations					-						
Estimating Quantity & Size											
Decimal Fractions						10					
Relationship of Common & Decimal Fractions						88					
Properties of Common & Decimal Fractions											
Percentages											
Proportionality Concepts						10					
Proportionality Problems						100					
2-D Coordinate Geometry											
Geometry: Transformations							-				
Negative Numbers, Integers & Their Properties											
Number Theory											
Exponents, Roots & Radicals											
Exponents & Orders of Magnitude											
Measurement Estimation & Errors											
Constructions w/ Straightedge & Compass							-				
3-D Geometry											
Congruence & Similarity								-			
Rational Numbers & Their Properties											
Patterns, Relations & Functions											
Slope & Trigonometry											
Number of topics covered by at least 67% of the A+ countries	3	3	7	15	20	17	16	18			
Number of additional topics intended by A+ countrie to complete a typical curriculum at each grade level	s 2	6	5	1	1	3	6	3 A+ countries			

In comparison (or contrast!), this is the topic coverage for a selection of socalled top-achieving countries. Just visually, what do you notice? [Solicit feedback.]

							and the second		ommon Core ontent Domains
К	1	2	3	4	5	6	7	8	9 - 12
				Geom	etry			Geometry	
Measurement and Data						Statis	tics and Prob	Statistics and Probabilit	
Number and Operations in Base Ten						The	Number Sys	Number and Quantity	
Ор	Operations and Algebraic Thinking					Expres	sions and Eq	Algebra	
Counting and Cardinality			Number	r and Oper Fractions		Ratios and Pr Relation		Functions	

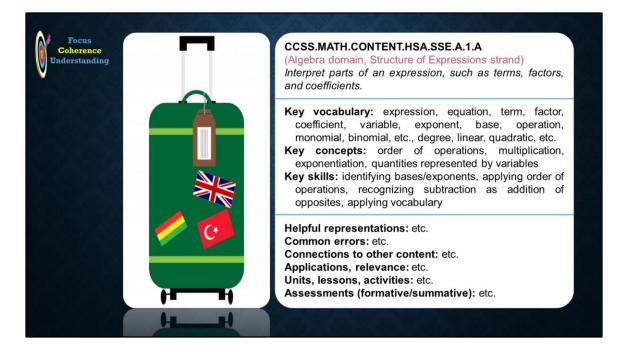
Now to talk briefly about the shift toward more coherence.

Here is another viewpoint on the various topics covered in the Common Core, organized according to larger categories that they call domains. You can see that there is a manageable number of domains with a gradual progression of topics in the early elementary years leading into topics in the late elementary years. For instance, there is a focus on measurement and data in K-5 (a topic which is actually often omitted in schools and districts), which then grows into a more sophisticated approach to data with studying topics in statistics and probability.



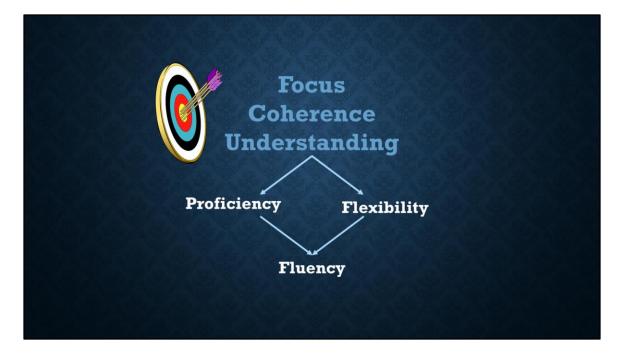
The CCSS in mathematics have been built around a set of learning progressions. Learning progressions are trajectories of learning, which is why I think of them like the arc of a ball being tossed, that move from elementary ideas to more sophisticated ones. The learning progressions in the CCSS are based on what is known from research about the sequence in which concepts and skills can be understood by students.

The website that I'm showing here, which is included on the Resources sheet in your folder, is a great site that contains an archive of the learning progressions in all of the content areas. I highly recommend reading them and using them in developing a scope-and-sequence or a curriculum map.



That doesn't mean that schools and teachers don't have work to do. The standards explain the basic content goals for students at the various age levels (although in high school, they leave it up to schools to decide on using integrated or traditional sequences). Schools and districts are doing additional work to interpret the standards, particularly as it relates to their particular context. Some people call this unpacking or unwrapping the standards. [Give a description of what this slide is attempting to convey.]

What I'd like to focus on for the rest of the session is understanding. We can come back to coherence and focus which are very important but often require a lot of work to understand how the pieces of the common core fit together in a later session if you are interested. They also involve schools and departments making clear commitments (I think) about what they would like their math programs to look like.



So this is your exit ticket. I'd love to hear from you--what you are currently doing to achieve the goals of focus, coherence, and understanding? Alternatively, what new things might you like to try? What overall reactions or questions do you have?

Resources

Common Core Math Standards: http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf Common Core progressions: http://achievethecore.org/page/254/progressions-documentsfor-the-common-core-state-standards-for-mathematics-detail-pg Achieve the Core resources: http://achievethecore.org/ Common Core writer (Phil Daro) against answer-getting, etc.: http://serpmedia.org/daro-talks/ Jo Boaler of Stanford on why we need Common Core: http://www.youtube.com/watch?v=pOOW0hQgVPQ "Why Do Americans Stink at Math?": http://www.nytimes.com/2014/07/27/magazine /why-do-americans-stink-at-math.html?_r=0 "Stereotypes that Distort Math Teaching/Learning": http://www.theatlantic.com/education/archive/ 2013/11/the-stereotypes-that-distort-how-americans-teach-and-learn-math/281303/ Common Core Tools: http://commoncoretools.me/ InsideMathematics (teaching videos): http://www.insidemathematics.org/ Edthena (observation tool): http://www.edthena.com/

Here are other useful resources [discuss as needed, as questions emerge].