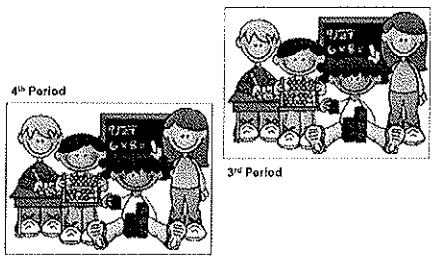
PHILADELPHIA AREA
MATH TEACHERS CIRCLE

Brownies and Word Problems

January 27, 2015

Once upon a time…
Your two 7th grade classes were well-behaved…



So you decided to bake them brownies!

But…while the brownies were cooling…
some mischievous scamp snuck in…
cut out a piece…and stole it!

The Brownie Problem

The tray of brownies must be split evenly between your two classes, and you have only moments to do so. Is it possible to divide the brownie tray evenly, using a single cut from your knife? Why or why not?

Questions

?

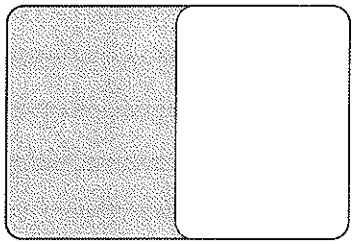
Assumptions (for now)...

- The tray is rectangular
- The missing piece is
  - Rectangular
  - Cut from *anywhere* in the tray
  - *Any* size
- Your cut is
  - A single cut
  - A straight line

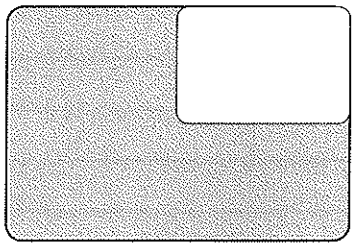
Other Considerations

Work in progress!!

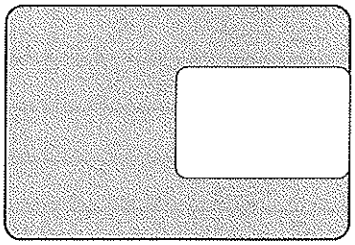
check back soon...



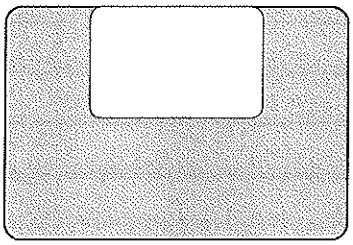
Hints: Simpler Cases?



Hints: Simpler Cases?



Hints: Simpler Cases?



Hints: Simpler Cases?

Teaching Tips

Ask students…

- To identify and try simpler cases
- To measure / cut
- To explain their thinking; in particular, if they think that drawing a line through the "middle" of the rectangle cuts the area in half, ask them to explain what they mean by "middle" and "how they know it cuts the area in half."
- To try multiple explanations, not just one (algebraic, geometric, etc.)

Extending the Problem

Can you solve the problem with a triangular piece/tray? Pentagonal? What shapes will work and what will not?

Why?

A General Solution

Any brownie tray that has *rotational symmetry* and any missing piece that also has rotational symmetry can be divided in half by a single, straight-line cut.

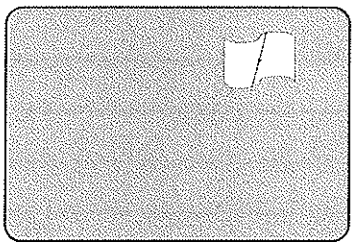
Why?

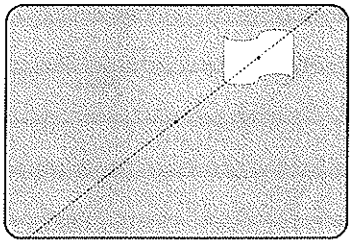
But First...

What about the *opposite*? In other words, are there any shapes that do *not* have rotational symmetry for which you can cut the brownie tray in half?

(The necessary condition)

Which ones? How? Why?





Some of the Common Core Connections

CCSS.Math.Content.5.G.B.3

Classify two-dimensional figures into categories based on their properties.

CCSS.Math.Content.6.G.A.1

Solve real-world and mathematical problems involving area, surface area, and volume.

CCSS.Math.Content.7.G.A.1

Draw construct, and describe geometrical figures and describe the relationships between them.

CCSS.Math.Content.8.G.A.1

Understand congruence and similarity using physical models, transparencies, or geometry software.

Problems with Word Problems

They are

- Easily misinterpreted (genre)

Esteban has 3 apples. Sandy has some apples, too. If Esteban and Sandy have 9 apples altogether, then how many apples does Sandy have?

Common answer: 9

See NOTE A (@END)

Problems with Word Problems

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- Not relevant (whales v. trolleys)

About how many dolphins are there in a blue whale?

SEE NOTE B

Problems with Word Problems

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- Contrived (MacGuffins)

Five years from now Mary will be twice as old as her sister; how old are the girls now?

SEE NOTE C

Problems with Word Problems

They are

- Easily misinterpreted (genre)
- Not relevant (whales v. trolleys)
- Contrived (MacGuffins)
- Acontextual & algrorithmic (street math)

Coconuts at a market in Recife, Brazil cost 35 cruzeiros each. How much are four coconuts?

SEE NOTE D

Sigh...so what can be done?

Word Problem 1

Do this on your own:

Identify the *numerator* and *denominator* of $\frac{6}{7}$?

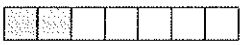
Word Problem 2

Do this on your own:

$$\frac{6}{7}+\frac{3}{7}$$

Word Problem 3

This is a "bar model" (or "fraction bar") for what?



Work with a partner, and use a bar model to find the sum and explain why it is the result:

$$\frac{6}{7} + \frac{3}{7}$$

Word Problem 4

You and your family brought 8 sandwiches to the school picnic. If you decided to share the sandwiches equally, how many would each person in your family get? Explain your reasoning.

Cognitive Demand

Procedures with Connections	Doing Mathematics
3	**4**
1	**2**
Memorization	Procedures without Connection

Cognitive Demand

Procedures with Connections	Doing Mathematics
routines & concepts	**explore & explain**
facts	**routines**
Memorization	Procedures without Connection

Examples

1) Use the order of operations to evaluate:

   $4 + 2(3 - 1)$

2) Using the numbers 1, 2, 3, and 4 *only once*—and the operations of addition, subtraction, and multiplication—what is the smallest positive number that you *cannot* obtain? How do you know?

   a) How many ways can you obtain 5?

   b) How do you know that you've got them all?

Examples

1) Solve the system of equations using *elimination:*

   $2x + 5y = 15$
   $-6x - 15y = -4$

2) A system of two linear equations has no solution. One of the equations in the system is $y = 3x - 4$. Give two examples of what the other equation in the system could be if the system has no solution. Give the equations in standard form. Explain how you found the equations.

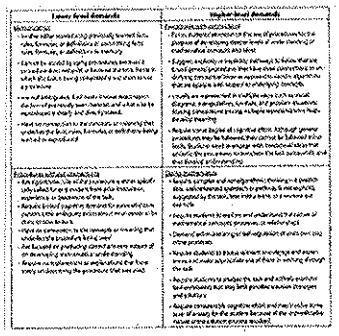
Examples

1) Find x:

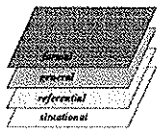
   $160(0.75)^x < 1$

2) Your baby brother accidentally ingests 160mg of aspirin at 8am, when you are babysitting. He is known to have a bad reaction to aspirin. The aspirin is metabolized at a rate of 25% each hour. Will there be any left in his system at 4pm—if 10mg in his system is enough to cause the reaction? Explain why or why not.

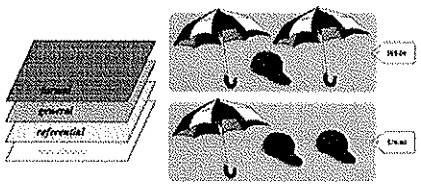
1) What is the formula for the area of a trapezoid?
2) Multiply (-5)(8).
3) Solve $2x + 4 = 18$ for x.
4) Find $\frac{3}{8} \div \frac{9}{8}$. ("Do not ask the reason why, just flip and multiply!")
5) Calculate the mean of {1, 4, 9, 11, 15}.
6) What is the *slope* of $y = 8x + 2$?
7) Multiply $(x + 2)(x - 4)$ using FOIL.
8) TRUE/FALSE: $8 + 2x^2 + 12x^3$ is a *cubic trinomial*.

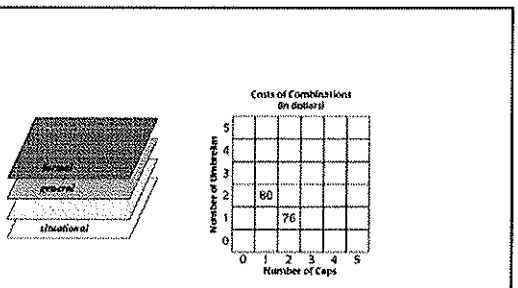


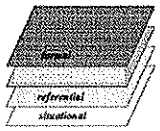
FOR LARGER VIEW, SEE NOTE E



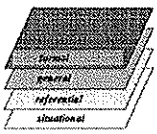
RME:
Realistic Mathematics Education





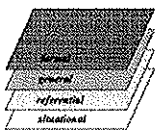


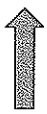
$$2U + C = 80$$
$$U + 2C = 76$$



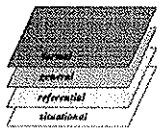
$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} E \\ F \end{pmatrix}$$

What we *should* do…





What we *often* do, instead…



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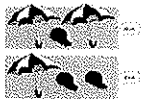


Our next workshop—
Tuesday, February 24th
5-7p (2501 Lombard St., TPS)
Usually the 4th Tuesday each month

NOTE A

Problems with Word Problems

They are

- Easily misinterpreted (genre)

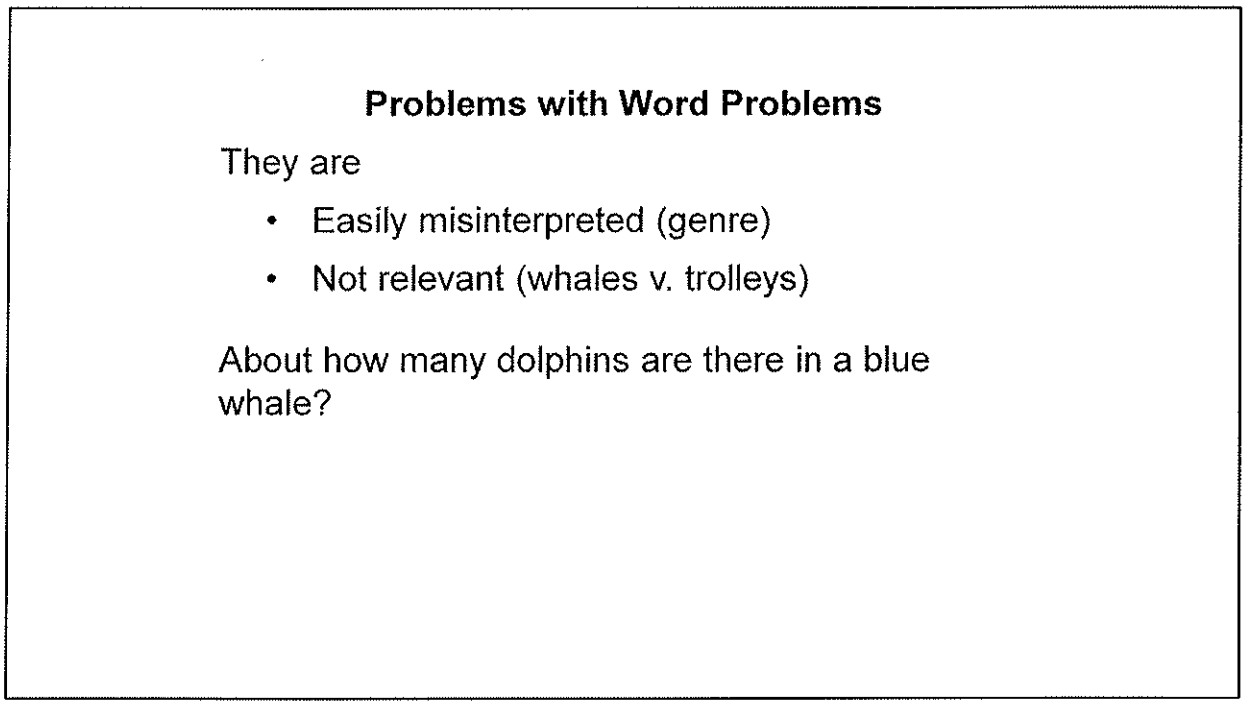
Esteban has 3 apples. Sandy has some apples, too. If Esteban and Sandy have 9 apples altogether, then how many apples does Sandy have?

Common answer: 9

We adults have learned to recognize this as a "math problem" (the genre of math problems) [a la Gerofsky] and that there are words within the problem that carry specific and important meaning. But children have not developed this ability (adults struggle with it, too). Often, the "joint ownership" of the 9 apples is confusing to students; the word "altogether" is often interpreted to mean "each"—as in, they *each* have 9 apples.

[D.D. Cummins, 1991; citing Decorte & Vershaffel, 1995, p. 19]

NOTE B



This problem is from an early-elementary unit on measuring lengths. Students learn about the length of an average dolphin and blue whale, and then are asked to make this comparison. In a research project in which I am very loosely affiliated [CBMP], students in urban areas were interviewed about their understanding of dolphins and blue whales and they were very confused by them. When these problems were changed to the lengths of bicycles, cars, buses, and trains, however, students were much more easily able to relate to (and understand) them. The idea is that the context of dolphins and whales was not relevant to students' lives in cities, especially those students who hadn't been to museums or aquariums.

NOTE C

Problems with Word Problems

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- Contrived (MacGuffins)

Five years from now Mary will be twice as old as her sister; how old are the girls now?

Students, upon seeing this question, have asked—"Why don't these girls know how old they are?" This is an example of a problem that no sensible person "would be caught asking" (Gerofsky, 1999). Other scholars have called problems like this "MacGuffins" (Wiliam, 1997)—which is a sort of nonsense term to indicate that the problem is nonsensical...there's no real good reason for even asking it!

[from Gerofsky, dissertation, p. 75; citing Marks, 1994, p. 610; also Wiliam, 1997]

NOTE D

Problems with Word Problems

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- Acontextual & algrorithmic (street math)

Coconuts at a market in Recife, Brazil cost 35 cruzeiros each. How much are four coconuts?

This problem is a documented, real-life situation. The twelve year-olds working in the markets in Recife were observed solving problems like this without aid of pencil-and-paper or calculator. Yet, when given the same exact problem on a worksheet in a classroom, the same children were unable to solve the problem. They tried to use the traditional method/algorithm for multiplication, but made errors in applying the steps; what's interesting, too, is that they were never told how to solve the problem—i.e., they didn't even *need* to use pencil-and-paper and an algorithm!

There are many similar stories. A junior high school teacher in California saw his students at a bowling alley and noticed that they were performing very complicated calculations to find each others' scores (and how many pins were needed to win, etc.). He gave them the same exact problems on a worksheet in his classroom, the next day, and the students couldn't solve the problems! The main ideas are that: school math problems are confusing because they are out of their usual context and because the methods that we often use to solve school math actually work against understanding the underlying concepts (and are easily forgotten or misused).

[Nunes, Carraher, Schliemann, 1993; also, Lave, 1988; also, Herndon, How to Survive in Your Native Land, 1971]

Lower-level demands	Higher-level demands
Memorization • Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas, or definitions to memory • Cannot be solved by using procedures, because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure • Are not ambiguous. Such tasks involve exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated. • Have no connection to the concepts or meaning that underlies the facts, rules, formulas, or definitions being learned or reproduced	Procedures with connections • Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas • Suggest, explicitly or implicitly, pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts • Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning. • Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.
Procedures without connections • Are algorithmic. Use of the procedure is either specifically called for or is evident from prior instruction, experience, or placement of the task. • Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done or how to do it. • Have no connection to the concepts or meaning that underlies the procedure being used • Are focused on producing correct answers instead of on developing mathematical understanding • Require no explanations or explanations that focus solely on describing the procedure that was used	Doing mathematics • Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example • Require students to explore and understand the nature of mathematical concepts, processes, or relationships • Demand self-monitoring or self-regulation of one's own cognitive processes • Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task • Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions • Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

NOTE E