WHO WILL SAVE ME?

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Supplies: Graph Paper

1. The Problem

The streets of Gridtown are laid out in a 17×17 grid, with east-west streets named 0^{th} Street through 17^{th} Street and the north-south streets named 0^{th} Avenue through 17^{th} Avenue. Gridtown has two fire stations, Station A at the corner of 2^{nd} Street and 3^{rd} Avenue and Station B at the corner of 15^{th} Street and 10^{th} Avenue.

Question 1 (*Do this in groups, but rather quickly gather the groups back together before moving on to the next question*)**.** If your house at the corner of 6^{th} Street and 10^{th} Avenue catches on fire, which station should send a fire truck to save you?

Leave the groups time to visualize the problem on their own grids.

The answer is actually Station B! The discussion of this question should center on the notion of *distance* in Gridtown. Since fire trucks cannot fly, they must drive along the roads. Therefore, the distance traveled by a fire truck from one corner to the next is measured by the number of blocks that they traverse! Your house is distance 11 (blocks) from Station A, but distance 9 (blocks) from Station B, even though a crow flying from Station A could get to your house faster (distance about 8.1 blocks vs. distance 9 blocks).

Now that we understand how distance works in Gridtown ...

Question 2 (*Set groups loose on this one, with extensions below handed out as appropriate*)**.** Draw a map that indicates which region of Gridtown Station A should serve and which region Station B should serve.

2. Hints

Ask groups to gain intuition on a smaller problem, though the smaller problem will not help them find the solution itself. Plotting points is not a bad idea!

Another method might be to ask groups to think about Euclidean geometry a bit: in this case, you would find the dividing line by constructing a

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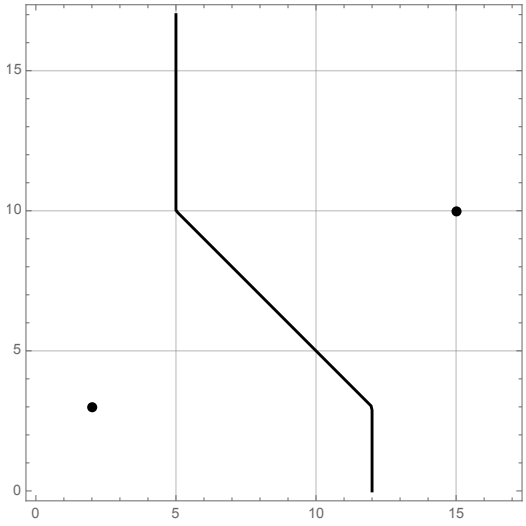
perpendicular bisector of the line segment $\overline{AB}$. The perpendicular bisector is the set of points where two circles of the same radius, one centered on A and the other on B, meet. What does a "circle" look like for our Gridtown distance?

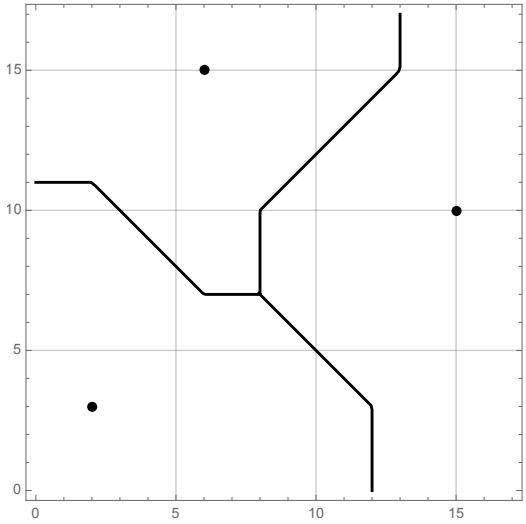
Here is the solution:



3. Extensions to the Problem

Question 3. How would your map change if the town built Station C at the corner of 6^{th} Street and 15^{th} Avenue?

Here is the solution:



For groups that get past the three-station extension, have them generate their own questions. Here are a few examples:

- What if east-west blocks were twice as long as north-south blocks? Vice versa?
- What if there were also diagonal streets in Gridtown?

4. Common Core Standards

The focus of this problem is on the Standards of Mathematical Practice:

MP1: Make sense of problems and persevere in solving them
MP2: Reason abstractly and quantitatively
MP3: Construct viable arguments
MP8: Look for and express regularity in repeated reasoning.

There are also connections to geometry:

5.G.A: Graph points on the coordinate plane to solve real-world and mathematical problems
HSG.MG.A: Apply geometric concepts in modeling situations