

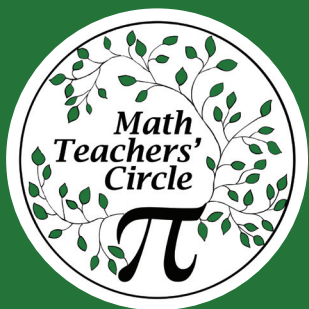
MTCircular

Winter 2014

THREE STORIES

BEST FOOT FORWARD

IMMERSION WORKSHOPS GIVE NEW
MTCs THE BEST START POSSIBLE



Ready for Exploration Sets, Planets and Comets
Learning Well, Teaching Well New Findings
Dispatches Nationwide MTC Updates
Problem Circle Puzzle Edition

Diving Headfirst Into Challenges

Dear Math Teachers' Circle Network,

Starting a new Math Teachers' Circle with an “immersion” workshop, in which teachers are immersed in doing mathematics for several days, is a challenge for a Math Teachers' Circle leadership team, but it is also a perfect opportunity to build a lasting mathematical community. Newly published research (p. 13) also provides evidence that MTC immersion workshops help teachers increase their mathematical knowledge for teaching. In “Best Foot Forward,” Hana Silverstein tells the stories of three new MTCs that held immersion workshops in Summer 2013.

Another challenge for Math Teachers' Circles is creating an environment where teachers can explore mathematics in a meaningful way without needing a lot of previous background knowledge. The AIM MTC recently conducted research on an open question during a MTC session and eventually published their results in the *College Mathematics Journal*. “Ready for Exploration” suggests some ideas for giving teachers a research or research-like experience within the constraints of the MTC model.

Joshua Zucker's “Problem Circle” should be especially appealing to puzzle enthusiasts this time, and mathematical game lovers will appreciate his exploration of the card games Set and Socks (“Game, Set, Match”).

We are very pleased to announce that Math for America will renew its support of the seed grant program for new Math Teachers' Circles this year. Two of the MTCs featured in “Best Foot Forward”—the Heartland MTC and the Southeast Ohio MTC—were Math for America seed grant recipients last year.

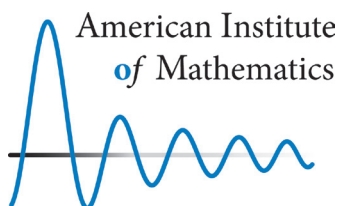
Finally, our website, mathteacherscircle.org, has a new look! One new feature we are especially excited about is our meetings calendar, where you can see what other MTCs around the country are up to. Thanks to all the MTCs who are participating in the calendar and the associated meeting surveys. In the near future, look for updates that will make the session resources section easier to navigate. Please also check out our new Facebook page at <http://www.facebook.com/mtcnetwork>.

As always, we invite you to share your MTC's news, resources, and photos with us so that we can help get the word out about what you are up to! You can reach us any time at circles@aimath.org.

Happy problem solving!



Brianna Donaldson, Director of Special Projects



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An immersion workshop is an important rite of passage for new Math Teachers' Circles. Often, this is a group's first chance to share with local teachers what Math Teachers' Circles are all about. It is also the first big event they will plan as a team.

According to three groups who held immersion workshops in summer 2013, it is well worth the effort. "The immersion experience is what creates community," said Bob Klein of Ohio University, one of the leaders of the Southeast Ohio Math Teachers' Circle in Athens, Ohio.

Gulden Karakok of Northern Colorado University, co-founder of the Northern Colorado Math Teachers' Circle in Greeley, Colorado, agreed. "The thing that's nice about immersion workshops is people really get to know each other, and they get relaxed with each other right away," Karakok said.

"When you go through something that's challenging and experience success together, I think that's what builds community," added Mark Brown of MidAmerica Nazarene University, co-leader of the Heartland Math Teachers' Circle in Olathe, Kansas. "Or maybe it's just the fact that you spend eight hours a day for five days together."

What follows are three different stories about the process of building a community grounded in a love of math and teaching.

SECOND-TRY SUCCESS IN COLORADO

The Northern Colorado Math Teachers' Circle is based in Greeley, Colorado, about fifty miles north of Denver. With a population of about 110,000, Greeley is home to four middle schools, five high schools, and a handful of charter schools.

Greeley is also home to the University of Northern Colorado (UNC), which serves as home base for the Math Teachers' Circle. Cathleen Craviotto, a former UNC faculty member, started the Circle. The Circle's leadership team also includes UNC faculty Gulden Karakok and Katie Morrison and local middle school teachers Bonnie Funk, Delia Haefeli, and Julie Samsel.

After participating in a "How to Run a Math Teachers' Circle" workshop in 2011, the leaders were fired up about starting a Math Teachers' Circle in their hometown and planned to kick off their Circle with a summer workshop in 2012.

Craviotto wrote a couple of grant proposals to raise money for the workshop. Within a few months, they

BEST FOOT FORWARD

IMMERSION WORKSHOPS GIVE NEW MTCs THE BEST START POSSIBLE

by Hana Silverstein

secured nearly \$15,000 from State Farm, as well as seed grants from the American Institute of Mathematics and the Mathematical Sciences Research Institute.

They had the money. They picked the dates and a tentative location. They were excited to start. Now they just needed teachers.

But only one person signed up.

“We weren’t able to get enough attention,” Karakok said. “We started sending flyers out and everything, but we weren’t getting any applications or interest by March or April. So we decided we probably should stop at that point and change our plan.”

The new plan: start small. Instead of kicking off with a summer workshop, they began by holding evening math sessions in fall 2012.

Now, there was a little more time to plan. The UNC Foundation helped the group create a budget. The organizers wanted to have the workshop off campus, far from the teachers’ homes. The dream location was Estes Park, known for its spectacular views of the Rocky Mountains, as well as for being home to the hotel in the movie “The Shining.”

Craviotto and her colleagues estimated \$40,000 for their dream budget. Teachers would stay in hotel-style lodge rooms on a YMCA campground. Since it was a

far-away location, gas money would be included, as well as full room and board. Door prizes, books, and other materials were also included in the budget.

At the same time, the leaders wanted to think about the possibility of not getting that kind of money. Could they still have a workshop? They came up with a back-up plan. They would hold the workshop on the UNC campus, and teachers could stay in dorm rooms. “Having those two plans helped us to see what we could include and what we didn’t need,” Karakok said.

They ended up with a little over \$30,000. “We were lucky that State Farm let us keep the money for the next summer. If we weren’t able to keep that money, we would have forced ourselves to get teachers there, but we were lucky. I think it worked out better.”

They redoubled their efforts on recruitment. “We started going to schools,” Karakok said. “We kept emailing and requesting time to meet with the school district administration. We drove around to the schools and introduced ourselves to the principals. That’s something I would definitely recommend to other MTCs—not just emailing, but face-to-face contact.” They coordinated with the school districts to make sure there were no conflicts the week of the workshop, and booked the dates at Estes Park.



Participants at the Northern Colorado MTC’s immersion workshop in Estes Park pose for a photo together at the conclusion of the workshop.



By the end of March, they had over 30 applications from all over Colorado, and even a couple from other states.

The leaders' priority was to accept teachers from the local school districts. After that, they invited applicants who lived close together, so even if they lived too far to attend academic-year meetings in Greeley or Denver, they could keep communicating with each other after the workshop.

"What do teachers do all summer?" wrote organizer Julie Samsel, afterwards, in *The Greeley Tribune*. "Well, 24 teachers from various parts of Colorado spent four days with me at a math retreat in Estes Park."

The teachers were immersed in problem solving from 8:30 a.m. to 5 p.m. for four days, stopping only for coffee breaks and communal meals.

After dinner each night, there was an optional evening session where teachers could keep working together or hang out and play math games. On other nights, there were social events like a barbecue outside on the campgrounds or an optional hiking trip.

Of course, four days are not enough to do everything. The leaders had some trouble picking problems and finding a focus. "One thing we probably want to modify for next summer is having our evening sessions focus more on how to implement these ideas in classrooms," Karakok said. "We did not have enough time to have teachers reflect back on how to do these things with their students."

Paul Zeitz of the University of San Francisco came to lead a few sessions. "He was one of the leaders at the AIM workshop," Karakok said. "He said he always goes and visits sessions. So, we contacted him and asked if he was available."

Zeitz described the kind of help he gives these new groups: "As little as I can. The trick is to give the least amount of help so that people will do things on their own. And it's because what you are teaching is not math, but a way of thinking about math where you learn to investigate."

Engaging with teachers from different backgrounds was a big learning experience. "We had fifth grade

teachers working alongside high school teachers," Karakok said. "Some had not seen geometry in a long time, or never had an experience with transformational geometry. One of the best comments we got on our post-evaluations was, 'We were actually thinking throughout the whole workshop.'"

"FUNSTRATION" IN KANSAS

The night before the Heartland Math Teachers' Circle workshop began, Mark Brown of MidAmerica Nazarene University was thinking about the fun and frustration the teachers would experience. In a moment of inspiration, he coined a new word: "funstration."

"I didn't think the word would become so prevalent during the week until Paul Zeitz heard it," Brown said. Zeitz, who had also helped at the Northern Colorado workshop, flew in mid-morning on the first day of the Heartland workshop, and after hearing the word, sent it into cyberspace to a group of leaders at AIM.

It was important for Brown and his co-leaders—MidAmerica faculty Gary Andersen and Lisa Erickson and middle school teacher Ashley Nuñez—to bring in outside help when planning the workshop. Based in Olathe, theirs is the first Math Teachers' Circle in Kansas.

They consulted with Zeitz, as well as Diana White of the University of Colorado Denver, one of the leaders of the Rocky Mountain Math Teachers' Circle. Both came and led sessions during the workshop.

A few months before the workshop, Brown went to the Circle on the Road in Puerto Rico, an MSRI-sponsored workshop for leaders of math circles. Being there gave him some great ideas for sessions.

Most of the funding for the workshop came from the Preparing Educators for Rural Kansas (PERK) Grant, funded by the U.S. Department of Education. "Part of the grant was for developing a separate middle-level program for preparing math teachers," Brown explained. "I was able to fit the MTC idea into

what we were already doing. So, we didn't have to go out and do a lot of meeting with people to try to raise money."

With money taken care of, the Heartland leaders considered their priorities. "We got a feel from the teachers, especially those that were parents, that they wanted to be home in the evenings. Originally, we had in mind to bring them to campus, looked at local hotels, but decided that wasn't a priority," Brown said.

"We were able to provide great lunches, one dinner out, snacks throughout the day. We went out for dinner on Tuesday night, and just had a really great time. I could tell the next day, the teachers had another level in their relationship with each other."

Many of the participants in the workshop at MidAmerica Nazarene teach at local private Christian and Catholic schools. "They don't have a lot of the same opportunities for professional development that public school funding has, so they found this to be something that's been really good for them."

Other recruitment efforts included doing three or four presentations at smaller workshops. They also sent out flyers to middle school teachers within 30 miles of Olathe, and used word-of-mouth to make connections at the private schools.

"We charged \$50 for the whole week," Brown said. "Even a small financial commitment lowers the probability of no-shows."

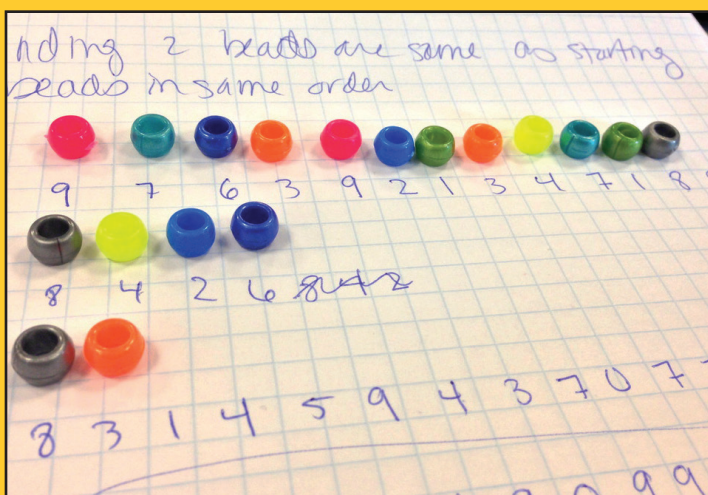
All 21 applicants were accepted. Most were middle school teachers. A few taught lower or upper grades. One husband-and-wife pair taught high school math and science.

The five-day workshop took place in the Bell Cultural Arts Center on campus, in a formal lounge area set up with round tables so teachers could work together in groups. Most days consisted of four one-and-a-half-hour math sessions, broken up by breaks and meals.

Zeitz set the tone after lunch the first day. "That was a highlight, the way he set the stage for getting everyone to believe they could be mathematicians, that



Scenes from the Heartland MTC's summer workshop at the Bell Cultural Arts Center at MidAmerica Nazarene University in Olathe, Kansas.



Scenes from the Southeast Ohio MTC's summer workshop at a new local Holiday Inn Express in Athens, Ohio. From top are pictured Operation Cookie Jar, math bracelets, and Pencils activities.

they're not always going to have answers; that's not necessarily the goal," Brown said.

"The environment was different from the Colorado group, but both groups were just as loose and had just as much fun," Zeitz said. "In both cases, what they had in common was the leaders of the Circles were really very generous. Not in a material way, but generous with spirit and time, and I think it rubbed off on the participants quite a bit."

Each of the team leaders led a math session. "Having enough energy was a challenge," Brown said. "I was excited each day, but it just took a lot out of me. I learned that when you do your first one of these, you should just be a participant, rather than leading a session yourself. Leading the sessions was what I wanted to do, and I enjoyed it, but it just made for shorter nights of sleep."

BRINGING MTC FEVER TO OHIO

Bob Klein of Ohio University, a co-leader of the Southeast Ohio Math Teachers' Circle, wants to establish a statewide network of Math Teachers' Circles.

"Ohio is all abuzz about MTCs right now," he said. His latest application to Ohio Board of Regents, which funded their 2013 summer workshop, requests additional funds to help start new circles across Ohio.

How does he have time for this? "I'm on sabbatical for the semester, but I don't have time for this," Klein said. "It's not so much that it takes time—it just takes excitement."

The Athens, Ohio-based team—which, in addition to Klein, includes middle school teachers Katie Hendrickson and Susan Matters, and math coach Nina Sudnick—brought a lot of excitement and a complementary set of skills to the planning of their immersion workshop.

Matters was largely in charge of recruitment. "We called her our wrangler," Klein said. "She was responsible for wrangling participants. She's in a number of school district and union organizations."

Sudnick, a former fourth grade math teacher who now works full-time as a math coach, helped make sure that the math sessions squarely targeted their intended audience. Hendrickson, meanwhile, created a website where teachers could apply for the workshop.

The group had help with the application process: the Stevens Literacy Center at Ohio University, which administered their grant, also took care of some logistics, including accepting and paying participants.

Participants who attended all four days of the workshop received a small stipend at the end. It was meant less as an incentive as a way to show respect that they were giving up their time.

They considered holding the workshop at a venue outside of Athens, but decided against it, thinking it would be too much to ask teachers to go out of town and leave their families during the summertime. Instead, they picked a new local Holiday Inn Express. Plus, by picking a low-cost venue, they could afford to spend more on food.

“AIM suggests that we try to treat teachers like kings and queens,” Sudnick explained.

Treating teachers well also meant having excellent facilitators and leaders. “Teachers are a pretty tough audience,” Sudnick said. “As soon as it gets boring, you’re going to lose them.”

Klein invited Judith Covington of Louisiana State University at Shreveport to lead a few sessions. A local magician came to do some math-related tricks. Hendrickson, who is working on her Ph.D., also led a session.

Klein also invited some professors from Shawnee State and Ohio State to lead a special session called “Ask a Mathematician.” Teachers were able to ask the professors how they became interested in mathematics, and what they thought about Common Core. “It gave us a chance to say, ‘What do you do? I teach kids math. How did you get into that profession?’” Sudnick said.

The leaders were mindful about tuning the content to their audience. “You don’t want it to be too easy or too hard,” Sudnick said. “Bob really worked hard at making math problems fun, but not a cakewalk. We talked about having the right amount of equilibrium there. We also tried to have problems that we could take back to the classroom.”

The workshop lasted four days. Each day included two problem-solving sessions, complete with pedagogy briefs, in the morning and in the afternoon, separated by an hour-long lunch. At the end of the day, there were wrap-ups and reflections.

One of the best sessions, called Operation Cookie Jar, was something Klein picked up from Gabriella



Photo: Colin Brough, stock.xchng

Pinter at the Circle on the Road in Puerto Rico. The problem involves 15 cookie jars, numbered consecutively from 1 to 15. The number of cookies in each jar is equal to the number of the jar. A “move” consists of choosing one or more jars, then removing one or more cookies from the chosen jars, but the same number of cookies from each jar. Teachers are asked for interesting math questions, and they explore the number of moves it takes to empty the jars.

“Our teachers just went nuts with that activity,” Klein said.

If the workshop has a lasting impact, Klein believes it is due to the immersion experience. “Math Teachers’ Circles are about collaborative problem solving, and collaborative problem solving in mathematics means putting yourself out there. It’s hard to expose yourself to what you don’t know, to take a risk. The immersion experience made it easier for all of us. That was four days well spent to develop community.”

The other thing it did, he added, was to break down walls of people’s perceptions of the divide between K-12 and higher education. “People would look at me and say, ‘I don’t get this problem. What do you think? Is this right?’ And I’d say, ‘Why don’t you ask your table? I don’t know. Why are you asking me? I’m doing the same problem.’”

Sudnick’s final words of advice, for people thinking of holding an immersion workshop, are to find educators who are jazzed up about math in their schools. “Who are your most engaged teachers? If you get them involved, they will help you engage other teachers.” 



Ready For Exploration

MTC Participation as a Research Experience

by Brianna Donaldson

MTC problems are mathematically rich, inviting multiple approaches and directions for investigation. As one participant in a recent “How to Run a Math Teachers’ Circle” workshop put it, “A good MTC problem opens up a mathematical landscape ready for exploration.” Some MTC problems even lead to open questions, questions that no one knows the answer to. Sometimes a session leader will point out open questions that lie further along the road that the MTC participants have started down. However, what about actually investigating one of these open questions during a MTC session?

In a recent session of the AIM MTC in Palo Alto, Calif., Brian Conrey and a group of teacher participants did just that. In the companion article at right, Conrey describes their experience, which led to a publication in the *College Mathematics Journal* and a press release from the Mathematical Association of America. But, one might argue, this was a special and perhaps unique set of circumstances. In general, is it

fruitful, or even possible, to attempt a research experience for teachers in the short time available during a MTC session?

Here, we will answer with a qualified “yes.” First, a caveat: MTCs are not a forum for extended research or research-like experiences for teachers. Although research experiences for middle and high school teachers still seem to be more common in the sciences and engineering, there are also research opportunities available for mathematics teachers, including through well-known programs such as PROMYS for Teachers and the Park City Mathematics Institute. These in-depth experiences tend to take place over a number of weeks in the summer, often in combination with intensive coursework, and most typically participation in such programs is of limited duration (with exceptions for teachers who go on to have leadership roles). By contrast, each MTC meeting tends to be fairly self-contained in terms of content. However, the same teachers may participate in MTCs indefinitely, with the

goal of creating a long-term professional community focused on a deeper understanding of what it means to do mathematics. In general, MTCs should be seen as a complementary activity to in-depth research experiences for teachers, just as MTCs complement and don't replace other valuable forms of professional development, such as lesson study.

That said, since MTCs focus so heavily on the practice of mathematics, why not occasionally spend a MTC session giving teachers a taste of what it's like to work on an open problem, or at least one that no one in the room, including the session leader, knows the answer to? "It's not necessary for the problem to be technically open for the teachers to have a research experience," points out Conrey. "If a problem is well known and open, then it is probably much too difficult. If it's not so well known, then it can be hard to determine whether it's actually open. What matters is for the solution and even the approach to be unknown to the session leader."

What other characteristics should a problem have to make it suitable for a MTC math research session? During the academic year, because not all participants can attend every meeting, the problem should not require much background knowledge. In addition, to make the best use of the limited time available during an academic-year math research session, the session leader should probably come prepared with a specific question. An immersion workshop presents a different sort of opportunity, since the same participants are in attendance for the entire workshop. In that case, it could be possible to have an introductory session early in the week, a follow-up session in which participants brainstorm questions and perhaps work on a few of the easier ones, and a further session in which participants spend time investigating one or more "open" questions that they have chosen with guidance from the session leader. A day-long workshop during the academic year could provide a similar experience for more seasoned MTC participants.

The session leader should also have an intuition that the problem is one that the group can make progress on, even if they don't completely solve it during the session. After all, the best MTC sessions of any kind leave the participants with more questions than they started with. However, as with any MTC session, it is extremely important to avoid creating unproductive levels of frustration. To help facilitate a productive



AIM Director Brian Conrey recalls the February 2011 session of the AIM MTC that eventually led to the publication of "Sets, planets, and comets" in the MAA's College Mathematics Journal.

We had previously known about the variation on the game Set where you look for two pairs of cards that both require the same third card to form a set. There are several other names for this variation, such as "SuperSet." We had come up with the name "Planet," because all the cards are in the same plane if you look at the game geometrically (see "Game, Set, Match," p. 18). What we were trying to figure out during the session was how many cards were needed to play the game Planet so that you're always guaranteed to find a planet. I had come up with this question ahead of time, but deliberately didn't try to think about it any further because I wanted the teachers to have a chance to investigate a question that no one knew the answer to. Just like we might do at an AIM research workshop, we split up into groups that worked on different approaches to the problem, looking at different configurations and trying to construct examples. I was really impressed by the enthusiasm of the groups as they worked! By the end of the night, we had figured out upper and lower bounds for the number of cards that guarantee a planet if you play with just the red cards, or with the full deck. Plus, one of us (Tom Davis) had begun writing code that would help us check for planets given a number of cards that was in between our lower and upper bounds. As the code came in over the next few weeks, we made note of any examples of nine cards without a set or a planet and began to suspect that such collections always summed to 0. We called the examples that summed to 0 "comets" and eventually verified that every single example of nine cards without a set or planet is a comet. This gave us the beautiful answer that if you play Planet with nine cards, you will always have a set, planet, or comet on the table!

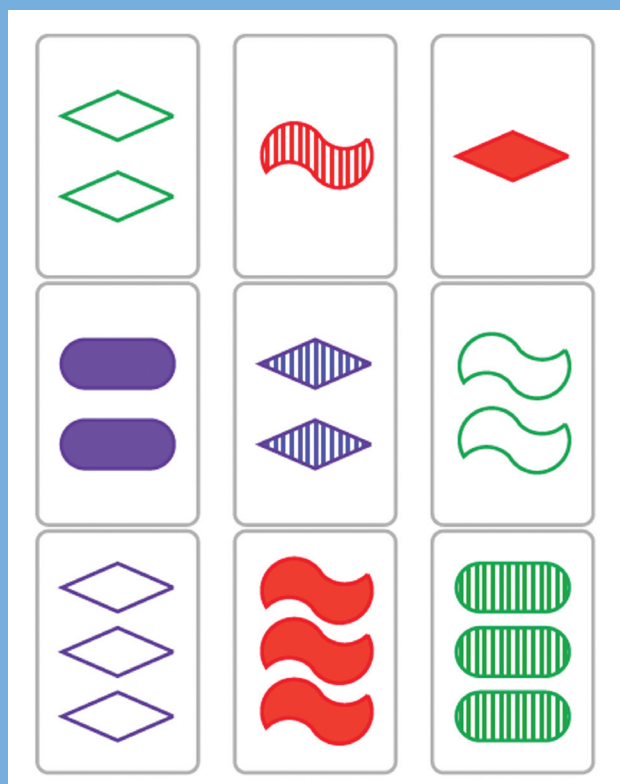
The Game of Planet

Nine cards are dealt. Players call out and claim sets (three cards that, in respect to each of the characteristics of number, shading, color or shape, are either all the same or all different), planets (two pairs of cards that require the same third card to form a set), and comets (nine cards that don't contain a set or a planet; see CMJ paper for full definition). The dealer replaces cards as they are removed.

In the example below, the middle and rightmost cards in the top row form a planet together with the leftmost card in the middle row and the rightmost card in the bottom row. That is, both pairs require a plain red oval to complete their sets.

If a player finds a planet containing a set, then they may only remove the set. The game ends when either the cards run out or there are no more sets, planets, or comets. The player with the most cards at the end wins. The AIM MTC proved that any collection of nine Set cards must contain a set, a planet, or a comet, so you should never have to deal extra cards.

Can you come up with a good strategy for finding planets?



atmosphere, Conrey advises choosing a problem that lends itself to investigation through examples: "A question with a concrete realization of the math that you're trying to do, like you have in a game, seems to work well." Julie Montgomery, a teacher who participated in the session, concurs: "The three-dimensional structure on which Brian positioned the Set cards allowed all of us to stand around in a circular fashion and to discuss possibilities and work them through to see if they were feasible. It felt like people's ideas just flowed freely and naturally and more people were able to jump in with ideas and respond to others' ideas."

The session leader should also rely on his or her own mathematical intuition to help organize the work that the group is doing. For example, in the Planet session, the AIM MTC split into smaller working groups that each focused on investigating a particular case or example, giving everyone a chance to contribute to the larger investigation. According to Leila Dibble, another participant, "It was exciting because no one seemed to have the answers, but together we tested out different ideas."

Finally, the problem should be approached with realistic goals in mind. In the case of the Planet session, the result turned out so nicely that the participants wanted to share their work (and the game of Planet) more widely, and eventually this led to the CMJ publication. As Montgomery says, "It is neat that math teachers can be involved in the evolution of innovative ideas and that we can participate to the point of publication, something we would not ordinarily (most of us, anyway) be able to do on our own." However,



Photo: Michal Mogmili stockxchng

while this outcome was exciting for everyone involved, publication should certainly not be a goal from the outset. Rather, a MTC math research session should be considered a success if the group makes progress on a problem that at the beginning of the session no one even knew how to approach. “One aspect of math that I love is the fact that, with a little tinkering, you can quickly find yourself in a problem space that has never before been explored,” says Avery Pickford, a participant in the Planet session. “This is what we did with Planet, and it is a great example of non-professional mathematicians doing real mathematics. I wish both teachers and students had more opportunities to do things like this.”

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MAA press release: <http://mathcomm.org/archives/exploring-the-mathematics-of-the-card-game-set/>

Learning Well, Teaching Well

Study: MTC Workshops Increase Mathematical Knowledge for Teaching

Teachers’ mathematical knowledge for teaching increases over the course of a four- to five-day intensive summer MTC workshop, according to a recently published study by Diana White, Brianna Donaldson, Angie Hodge, and Adam Ruff. Teachers who attended workshops at three sites around the U.S. participated in the study. On the first and last days of the workshops, teachers completed the Number Concept and Operations portion of the Learning Mathematics for Teaching instrument, developed by researchers at the University of Michigan to measure teachers’ mathematical knowledge for teaching. On average, teachers’ mathematical knowledge for teaching scores increased by .34 standard units over the course of the workshops. This increase was comparable to the results reported by another study in which teachers took a 48-hour professional development course that was focused on number concept and operations. By contrast, the MTC workshops involved fewer hours, covered multiple topics in addition to number concept and operations, and did not specifically focus on topics covered by the Number Concept and Operations instrument. Previous research has found a positive correlation between teachers’ mathematical knowledge for teaching and student achievement scores (Hill et al., 2007). The paper by White and colleagues is the first published

article from an ongoing study that MTCs around the country have generously participated in each summer since 2010. The article appears in the *International Journal for Mathematics Teaching and Learning* and is freely available from the journal’s website.

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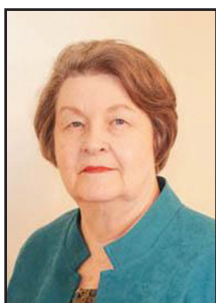
For links to these resources and more, visit us online at <http://mathteacherscircle.org/newsletter>.

Katie Hendrickson of Southeast Ohio MTC Wins Buck Martin Statewide Teaching Award



Katie Hendrickson, co-founder of the Southeast Ohio MTC and a seventh-grade math teacher at Athens Middle School in Athens, Ohio, was honored with this year's Buck Martin Award for Exemplary Mathematics Teaching by the Ohio Council of Teachers of Mathematics. This statewide award is given to one secondary teacher in the state of Ohio each year. To be considered for this award, the teacher must have previously been chosen as the outstanding secondary mathematics teacher in one of the eight districts in Ohio. Read more at http://www.ohioctm.org/teacher_awards.htm. Learn more about the Southeast Ohio MTC at <http://seomtc.weebly.com/>. 

Charlotte McShea Named Mississippi College Mathematics Teacher of the Year



Charlotte McShea of the South Mississippi MTC, Professor of Mathematics and Chair of the Department of Mathematics at William Carey University in Hattiesburg, Miss., was recently named College Mathematics Teacher of the Year by the Mississippi Council of Teachers of Mathematics. McShea was also a 2012 honoree of the Higher Education Appreciation Day—Working for Academic Excellence program, which was established by the Mississippi Legislature to recognize outstanding students and faculty in Mississippi's colleges and universities. During the presentation of this award, the presenter mentioned quotes from recommendation letters written by Dr. McShea's students. One letter said, "Dr. McShea has been, and still is, a mentor to many former students who have become math teachers." Read more at <http://www.wmcarey.edu/news/wcu-professor-named-mathematics-teacher-year#sthash.Gu8299FL.UFsDCdNQ.dpuf>. 

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A Warm Welcome to Our Newest MTCs

The nationwide network of Math Teachers' Circles is growing. Each of these groups sent a leadership team of about five people to one of our Summer 2013 workshops on "How to Run a Math Teachers' Circle." It is expected that they will each start their own MTC by Summer 2014. These new MTCs will join approximately 55 active MTCs and several others in the planning stages across the United States and its territories. A warm and hearty welcome to the Network's newest members:

- New Haven, CT: New Haven County MTC (<http://tinyurl.com/newhavencountymtc>)
- New London, CT
- Boca Raton, FL
- Des Moines, IA: Iowa Heartland MTC (<https://sites.google.com/site/iowaheartlandmtc/home>)
- Cambridge, MA: Boston MTC @ Harvard
- Marquette, MI
- Sault Ste. Marie, MI
- Gulfport, MS: South Mississippi MTC
- Missoula/Billings, MT: Montana MTC
- Newark, NJ
- Columbus, OH: Columbus Math Circle (<http://columbusmathcircle.wordpress.com/>)
- Oklahoma City/Tulsa, OK: MTC OK (<http://www.hisparks.com/mtcok.html>)



Visit our website at <http://www.mathteacherscircle.org/> for an interactive version of this map.

Helpful Resources Now Available Online

Exploding Dots, and More, Online

James Tanton, Mathematician-in-Residence at the Mathematical Association of America, has recently published an online video course about a topic that has grown to be one of the most popular Math Teachers' Circle sessions ever: Exploding Dots! The course is one of many resources available at Tanton's site <http://gdaymath.com/> and can be viewed at <http://gdaymath.com/courses/exploding-dots/>.

The Collaborative Mathematics Project

Jason Ermer, a co-founder of the MTC of Austin, has recently started the Collaborative Mathematics project (<http://www.collaborativemathematics.org/>), an innovative online project in which middle and high school students from around the world are encouraged to work collaboratively on problems posed by Ermer in challenge videos. Ermer said, "My work with the MTC community was a pivotal influence in my starting up the project." 

Dispatches from the Circles

MTC Facilitators Offer Local Updates from Across the Country

California •

The San Benito County Math Talks, based in Hollister, Calif., received grants from the National Association of Math Circles (NAMC) through MSRI, the desJardins/Blachman Fund through AIM, and the Monterey Bay Area Math Project.

– Gloria Brown Brooks

Hawai'i •

Math Teachers' Circle of Hawai'i (MaTCH) has received another year of funding from the state-allocated NCLB funding under ESEA IIA, and has experienced much success recruiting teachers. In the fall we had a total of 57 teachers from elementary through high school, including a few from neighboring islands. We are now faced with a room capacity issue and are exploring the option of e-table technology. In January we will be collaborating with colleagues from the Education Development Center, Inc., to train a few of our teachers and develop this technology for expanding MaTCH.

– Linda Venenciano

Louisiana •

The North Louisiana Math Teachers' Circle has secured matching funds from the Educational Advancement Foundation and the Noel Foundation, Inc., to fund our circle through May 2015, including funds to allow James Tanton to be the leader of our final spring meeting in May 2014. Also, we have ordered t-shirts for our group with our organization's new logo designed by one of our members. [Editor's Note: Circle leader Judith Covington was also promoted to full professor this year.]

– Judith Covington



Mississippi •

Beth McShea and Joni Cooks of the South Mississippi MTC led a math circle at the Mississippi Council of Teachers of Mathematics Fall Conference. MTC Leader Beth McShea just received notice that she is now a National Board Certified Teacher in Adolescent and Young Adult Mathematics. [Editor's Note: Also, Charlotte McShea was named College Mathematics Teacher of the Year by the Mississippi Council of Teachers of Mathematics (see p. 14).]

– Charlotte McShea

Ohio •

At the October 2013 meeting of the Ohio Council of Teachers of Mathematics, Ohio's two original Math Teachers' Circles, the Cincy Math Teachers' Circle and the Southeast Ohio Math Teachers' Circle, joined together to offer participants two "mini-circle" sessions on the game of Set and Conway's Rational Tangles in order to generate further interest in MTCs. As a result, two new teams have promised to submit applications to attend AIM workshops and begin developing their own Circles in northwest and south-central Ohio.

– Bob Klein and Stephan Pelikan

Oklahoma •

The Math Teachers' Circle of Oklahoma in Tulsa, a new Circle after last summer's workshop, received a \$2,000 seed grant in October from the Mathematical Sciences Research Institute (MSRI). We were fully funded for our inaugural MTC session on October 3 by the Math Department and the College of Engineering and Natural Sciences at The University of Tulsa (TU). And the TU Math Department funded our 2nd meeting on November 7 as well. Both meetings were well attended and received great reviews by all. So, we are off to a very good start in Tulsa!

– Marilyn Howard 

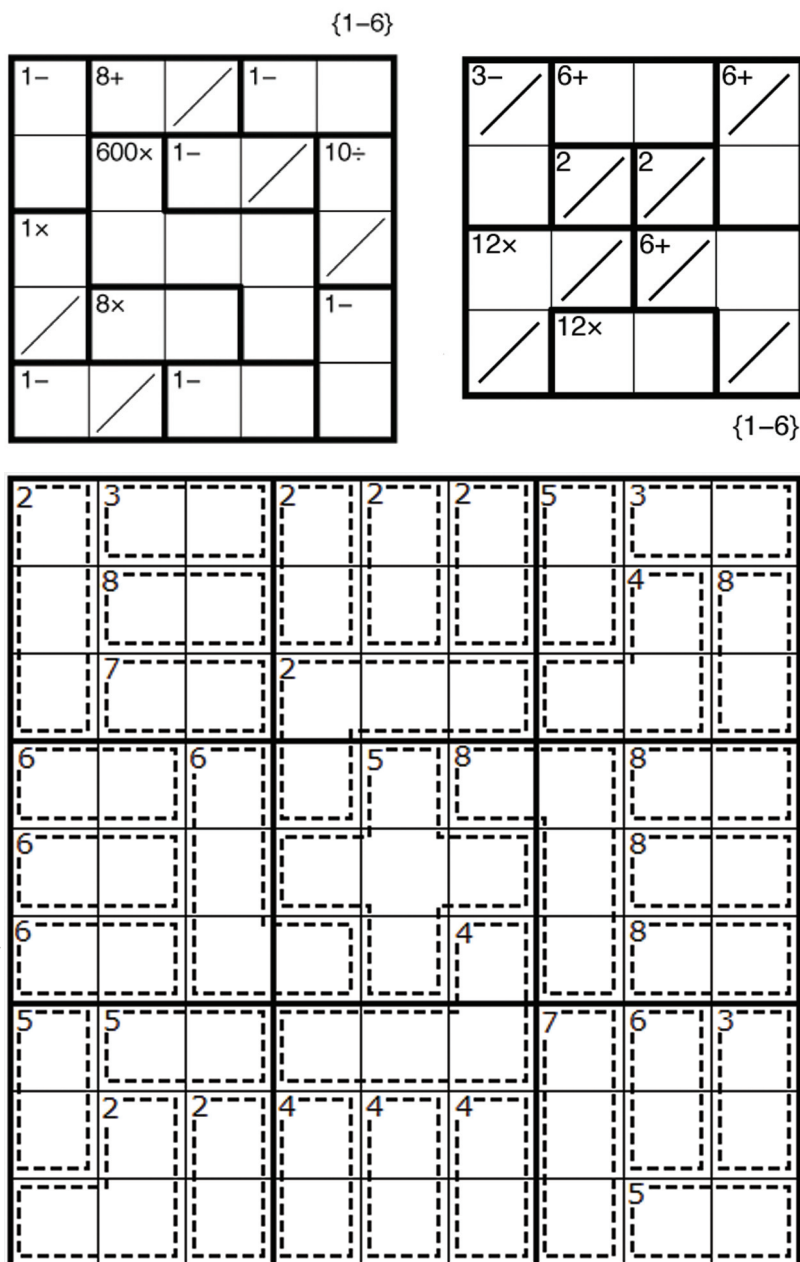
Problem Circle: Puzzle Edition

by Joshua Zucker

I know a lot of middle school students practice KenKen® puzzles to exercise their logic while getting a little bit of arithmetic review. The two puzzles at right, a recent invention of Thomas Snyder, have similar rules: use each digit from 1 through the number of spaces in a row, once in each row and once in each column, so that in each cage the indicated operation gives the result shown. But these Tom/Tom puzzles have a little twist: some of the cells take two digits, on the top and bottom of a fraction bar! This opens up lots more possibilities, and gives students (and their teachers!) some more interesting and challenging arithmetic practice.

Many of us have solved sudoku, placing the digits 1 through 9 once in each row, column, and box. Some of us have done killer sudoku, where the clues give the sum of the cells enclosed by dotted lines. But Bram de Laat provides us a new twist on that: the clue in the corner is a divisor of the sum, so when you see a 4 there, the sum of the digits could be any multiple of 4. Also, one surprisingly useful rule: digits cannot be repeated within a dotted region, either. Warning: this one takes a lot of patience and methodical searching through possibilities, as well as a bit of mathematical insight.

If you think this is fun, you should also try the [Sum Relations Sudoku](http://puzzleparasite.blogspot.com) at Bram de Laat's blog, *Puzzle Parasite*, another great sudoku with some nice mathematical thinking in it. Have fun! 📖



Puzzles: Factory Killer Sudoku by Bram de Laat, <http://puzzleparasite.blogspot.com>. Tom/Tom on left, by Thomas Snyder, originally posted on <http://gmpuzzles.com> (07/30/13). Tom/Tom on right, by Thomas Snyder, written as a sample puzzle for the 2013 U.S. Puzzle Championship, <http://wpc.puzzles.com>. Posted on <http://gmpuzzles.com> (06/19/13), where you can also find the puzzle from the actual championship test.

For more links and resources, including answers to our last problem, visit us online at <http://mathteacherscircle.org/newsletter>.

Game, Set, Match

by Joshua Zucker

Mathematical games make excellent Math Teachers' Circle sessions: they are engaging for teachers, and can often easily be passed along to their students. They are usually accessible in a wide range of ways and at different levels of depth, so mathematicians can appreciate them in one way while schoolchildren can appreciate them in a different way. Some games are more obviously mathematical, or mostly played only by mathematicians (such as Nim), while other games are already likely to be well known to most teachers and quite a few students.

The game of Set is a wonderful example. It is widely played in classrooms around the country, with a surface level involving lots of pattern recognition and a challenging, but not too difficult, amount of abstraction. Kids can often outperform mathematicians, so understanding of the deeper theory gives a deeper appreciation of the game but (unlike Nim) not necessarily an easy forced win over people who don't have that knowledge. The diversity of questions, and areas of mathematics that are touched in exploring them, also make Set an excellent choice.

I have seen several different versions of this session from prominent MTC leaders, including Brian Conrey, Judith Covington, and Diana White. The first part of these sessions always involves a brief introduction to the game, in which people who have seen and played it before help mentor teachers who are new to Set. They also often open with some time to get a shared vocabulary for describing the cards, and perhaps a little practice of the idea that given any two cards, there is a unique third card that forms a set. That third card is defined as the one such that, for each of the four characteristics (number, shading, color, and shape), the three cards are either all the same or all different. Take, for example, the first row of the diagram on the next page. With two solid red squiggles and two striped

purple diamonds, the card to form a set must have the same number, two, but will be different in each of the other characteristics, hence plain, green, and oval.

There is also often a brainstorming session in which participants generate questions that interest them. How many different sets are in a deck? For that matter, how many cards are in a deck? What is the probability that three random cards form a set? What is the probability that 12 randomly dealt cards contain at least one set? For those familiar with the puzzles, how does one generate a collection of 12 cards with exactly six sets? Some sessions choose to tackle these questions.

Probably the most interesting innovation in these sessions, though, is the technique for shifting the perception of the Set cards to expose the underlying geometric structure. This is the game that Conrey calls "Set Collect." To play, begin with some cards on the table, and the rest of the deck in your hand. If you hold the third card that forms a set with two cards already on the table, you can play that card from your hand. The object of the game is to try to get as many cards as possible placed on the table.

If you begin with two cards on the table, there's only one third card that goes with them, and then the game is over. Any two cards on the table will form a set with the third card that is already there, so you cannot play any more cards.

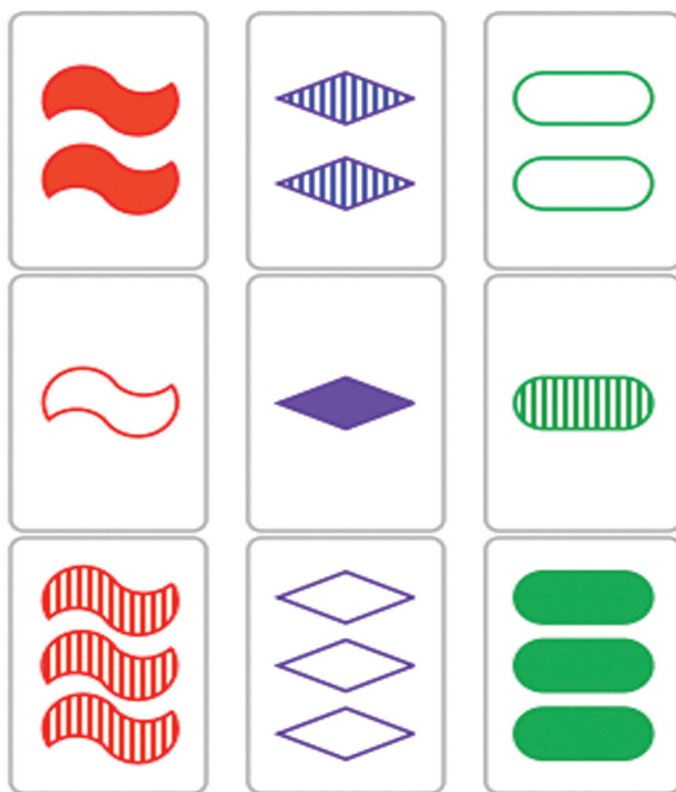
On the other hand, when you begin with three cards that do not form a set, things quickly become more interesting. Each pair of cards will call for a different third card. And then those third cards start to pair up with each other as well! After some exploration, you'll discover that you get stuck with nine cards on the table now. Perhaps you'll find a useful way to arrange them in which you can see the sets that have been formed, as in the example on the next page, where

the first two cards of the top row and the first card of the second row are the three cards that were on the table at the beginning. As you can see, when the cards are arranged in this manner, each row is a set, each column is a set, and each diagonal is a set, including the “wrap-around” diagonals!

Now we have the basis for making the transition to viewing Set as a geometric game. Each card is a point. Each set is a line. And we have discovered the way a line, plus one point not on the line, forms a plane of nine cards! This naturally leads to more questions: how many planes are there in the deck? Can two planes intersect in just a line? In just a point? Given a plane and a point, is there a plane through that point parallel to the first plane? What does parallel mean, anyway? Is there more than one plane through that point that doesn't intersect the original plane?

Perhaps the most natural next question is what happens to this collection game if you play it beginning with four non-coplanar cards. Once we get beyond a Set plane, how can we arrange all the cards we collect? What about a fifth card?

The Collect game leads to excellent discoveries in a number of other games as well. You can play a version of it with [Socks](#) and learn a lot about its mathematical structure. In Socks, each card has images of one or more different socks, and you need to make a set of exactly three cards such that all the socks come in pairs. When you start playing the Collect game with two cards, you find a unique third, just as in Set. But with Socks, when you start with three cards, you find you end up with a collection of seven that organize themselves nicely in a Venn diagram or a projective plane! This leads to some newly interesting questions: Can the deck of 63 cards be split into collections of seven, in the same way that the Set deck of 81 cards splits into collections of nine? What happens if you



take those seven and add another card to continue playing the Collect game?

There's a deeper connection, too, in the ways that both these games have the idea of “three cards go together if they add up to zero.” You need to find the right sense of adding up to zero, though!

Even though a lot of these questions and answers won't help you become a better Set player, they do spark a lot of curiosity and set the foundations for future explorations in linear algebra, affine geometry, discrete geometries, and even some number theory as you can find in the Set session notes on the MTC website. Enjoy! ☐

For links to resources related to this story, visit us online at <http://mathteacherscircle.org/newsletter>.



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