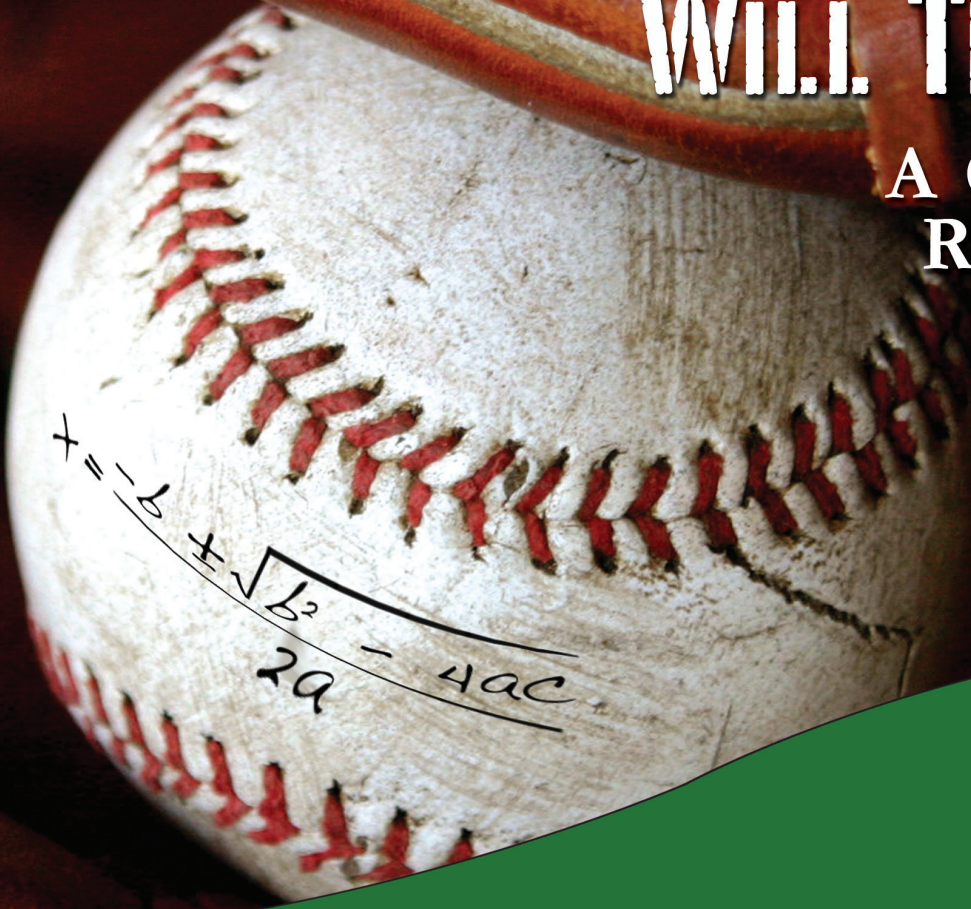


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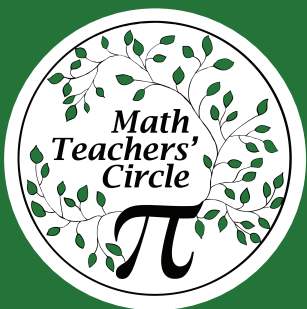
Winter 2013

IF YOU BUILD IT, WILL THEY COME?

A Game Plan For
Recruitment and
Retention of
Members



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$



The "Good Problem" Problem Classic Topics
Learning to Lead A Participant's Perspective
Simple, Or Impossible? Problem Circle
In Session Ron Graham's Sequence

Creating Communities of Mathematical Practice

Dear Math Teachers' Circle Network,

The Conference Board of the Mathematical Sciences recently recognized Math Teachers' Circles as “communities of mathematical practice in which teachers and mathematicians can learn about each others' profession, culture, and work.” Building and maintaining these communities takes constant effort and attention. Inspired by a great discussion on the Teachers' Net Google Group earlier this fall, this issue's feature, by Jessa Barniol, focuses on recruitment and retention of teacher members and describes some ways that MTCs have successfully created communities that are valued by teachers and mathematicians alike.

The mathematical heart of MTCs is a focus on investigation, and so I am particularly pleased to draw your attention to recent research by a team at University of Colorado Colorado Springs. Among other results, teachers used significantly more inquiry-based teaching practices and demonstrated significantly increased conceptual understanding of mathematics after one year of MTC participation.

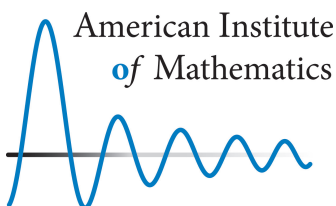
For the past three years, we have been asking workshop participants at “How to Run a Math Teachers' Circle” workshops to tell us what they think makes a good MTC problem (we also ask what makes a good MTC session). Joshua Zucker synthesizes some of the key ideas that have emerged from these always-interesting discussions, integrating them with his own and with references to a wonderful set of “classic” MTC session notes that I encourage you to explore.

Instead of a featured teacher and MTC this time, we drew on the activities of Member Circles to bring you two other pieces: a lively blog post from Fawn Nguyen, mathematics teacher at Mesa Union Junior High and member of the Thousand Oaks MTC's leadership team, about her team's experiences at a “How to Run a Math Teachers' Circle” workshop last summer; and a featured session co-written by Joshua Zucker and Susan Holtzapple, mathematics teacher at Cupertino Middle School, based on their experience co-planning and leading a session of the AIM MTC earlier this fall.

Happy problem solving!



Brianna Donaldson, Director of Special Projects



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IF YOU BUILD IT, WILL THEY COME?

A Game Plan For
Recruitment and
Retention of
Members

by Jessa Barniol

If you build it, they will come. Right? But what if you threw a party and no one came? What if you set up a Math Teachers' Circle meeting complete with a great topic and session leader, and no one turned up?

It's a scenario that Circle leaders dread. And although it's highly unlikely any group will ever garner a completely null set of attendees, a variety of recent factors, including low morale, teacher layoffs, other professional development requirements and the shortage of spare time faced by overworked teachers, have contributed to flagging attendance in many Circles across the nation.

A recent discussion on Teachers' Net, the online community for members of the MTC Network, garnered a few great suggestions to respond to low attendance. For instance, Diana White of the Rocky Mountain Math Teachers' Circle in Denver pointed out that the Common Core State Standards are currently the priority in teacher professional development and Circles should make this a well-advertised priority as well. Michelle Manes of MaTCH (Math Teachers' Circle of Hawai'i) mentioned that her teachers are given the choice between professional development credits or a stipend, thanks to grants. Steve Pelikan of the Cincy Math Circle in Ohio brought up the importance of taking time at the end of a session to reflect on what was accomplished and what was learned, to talk about classroom connections, and to think about how the next session could be even more productive.

But what else can be done? Here are some more tips and tricks from some of the most successful groups in the Network in terms of recruiting new attendees and hanging on to prior attendees. Their advice follows three main themes: build the Circle well from the ground up, remain in constant contact with the group, and be sure give the teachers what they need from the Circle.

BUILD A COMMUNITY.

"Your best asset for the survival of your group are its existing members: teachers who have attended, enjoyed themselves, and hopefully, returned for more," Michael Nakamaye said. He is the leader of the Albuquerque MTC, a group that regularly hosts around 25 teachers per meeting. Nakamaye said that about two-thirds of the teachers are recurring participants, regulars he calls the "core group."

Nakamaye said he always encourages members of the core group to bring colleagues with them to meetings.

"People are more likely to come regularly if they're coming with a fellow teacher from their school, someone who can talk them into coming after a long work day," Nakamaye said.

However, he also stressed the importance of building relationships between participants from different schools, particularly between the involved teachers and the faculty mathematicians involved.

"There should be time to mingle, to circulate and meet new people," Nakamaye said, explaining that his group's meetings, as in many Circles, often begin with a meal and time to socialize and unwind before getting down to the math. "Then your group becomes more like a community and less like a lecture hall."

WORK TOGETHER TOWARD A COMMON GOAL.

Judith Covington, leader of the North Louisiana Math Teachers' Circle, also stresses the fact that community is important within the Circle. The most important step, she said, is to ensure that your Circle is something you are doing "with" your participants, rather than "to" them or "for" them.

"On a recent evaluation, one teacher wrote that as something we were doing right," Covington said. "We never quite thought about it in those terms before, but it really is the best approach. It should be the group setting out to learn together, rather than a presenter setting out to talk at them for an hour."

SOLICIT FEEDBACK AND TRY NEW THINGS.

Keeping with the theme of working together with the members, many group leaders recommend surveying members for input on how to run the Circle, particularly which day and time would be most convenient for them to attend. When Nakamaye's group responded that they were interested in trying a Saturday morning meeting, he couldn't believe it, but decided to try it anyway. Just in case no one were to show, he scheduled himself as that particular session's leader, and to his surprise, the session was as much of a success as the group's regular Thursday evening meetings, both in terms of attendance and teacher involvement.

Covington's group, meanwhile, has overwhelmingly and repeatedly voted against any weekend commitments, and their group meetings remain firmly rooted

in Monday evenings, as something to look forward to at the start of the week. Meanwhile, the Math Teachers' Circle of Austin, led by Altha Rodin, meets on either Thursdays or Wednesdays, changing at the request of surveyed group members.

SPEND SOME QUALITY TIME TOGETHER.

What if you are just beginning your group and don't have access to an existing group to survey or invite to meetings?

Rodin said that the best way to launch a Circle is the way the MTC Network recommends: with a residential overnight workshop for teachers, for several days in the summer. "This will give you a tight-knit community, a group of regulars, and a firm footing to start," Rodin said. Over half of the participants of the MTC of Austin's first workshop in 2010 are still among the group's regular participants.

BEGIN WITH A BANG.

Covington's group did not begin with a workshop, but rather a large party-like event in which she and her group's other organizers pulled out all the stops.

"We had door prizes, gift cards, great food, and a minor celebrity of our state's mathematical community as our first session leader," Covington said, adding that this approach could also work for the first meeting of the year or semester to inject life into a languishing group.

"Advertise like mad, show the teachers what your group is all about, and collect the email addresses of all the attendees," Covington said. "Then you have a great starter list to begin to invite to regular meetings." With this approach, the North Louisiana MTC attracted 45 attendees at their first meeting in 2010, and they have since leveled off to around 25 attendees per meeting.

In fact, giveaways, freebies and treats are a great way to garner ongoing attention, Covington said. For example, for an upcoming session involving Polydron geometric construction sets, the group plans to buy a set for every teacher in attendance to use during the session and then take back to their classrooms. Her group's organizers hope that the promise of these in-demand classroom supplies will garner unusually high attendance for that session.

Similarly, Rodin said, creating lesson plans based on each session for teachers to take directly back to their own classrooms is a really great way to add value to the meetings and give teachers something they can really use, on top of the fun and personal math knowledge they already get from the sessions.

REMAIN IN CONSTANT CONTACT.

Rodin also said that a good rule of thumb is to have a potential group list about ten times bigger than the number of teachers you actually want at your session, particularly when working within an urban setting. She regularly reaches out to a list of 200 people in order to glean 20 to 30 participants for each meeting.

"The trick is to try for a steady stream of teachers, rather than the same exact group of teachers, at every meeting," Rodin said. "Our aim is to always have around twenty teachers, but many of the teachers can only attend one or two sessions per year. Teachers are tired; they have a lot on their plates. It's best to just get used to the reality that they can't all attend every single meeting, and you can't expect them to."

Covington also stresses the importance of maintaining your own email list to keep in constant contact with prior participants. "People who have come before are more likely to come again, even if infrequently," Covington said. "I never delete an email address, but I do include an opt-out message at the bottom, just in case. Teachers rarely opt out, but it's just common courtesy and, in some cases, required by law. So keep that in mind."

Covington also recommends selecting your meeting dates a year or semester in advance, and at the first meeting, giving each attendee a flyer with all the dates for whole year. "Then send the schedule to your whole list and post it online," Covington said. "That way, the teachers can plan ahead well in advance." Covington also mentions it's valuable to keep reminding teachers, especially those who have already registered, of the upcoming meeting. "I usually send a reminder email a day or two before so they don't forget," Covington said.

THINK OUTSIDE THE INBOX.

Nakamaye agrees that maintaining a good email list of prior and prospective participants is important to the success of a Circle. "That contact is important

to remind them of your group, even when they can't make it," Nakamaye said.

However, Nakamaye points out that you can sometimes get a lot farther with people with something tangible.

"Computer communication has lost some of its value in modern society," Nakamaye said. "People are so bombarded with messages, email in particular, that email contact is getting to a point of diminishing returns. It isn't always the most effective way to reach people." His group initially reached out with a good quality printed brochure in teachers' school mailboxes and received a great response to it. This approach is particularly useful when, as in Nakamaye's region, mass emails to teachers are not permitted unless it's with your own list.

Rodin, meanwhile, also realizes the limited potential of relying on her own email list and relies on the additional help of a point person with a wider sphere of influence. Her particular point person is Susan Hemphill at Region 13, a local branch of the Texas Education Agency. Hemphill is able to cover a wider base of interested teachers, forwarding messages and personally inviting teachers she knows who might be interested. Rodin attributes much of her group's success to Hemphill's involvement.

Covington, similarly, said she intentionally chose teachers from each of the three neighboring parishes (the Louisiana equivalent of a county) to be members of her Circle's leadership team, specifically so the Circle would have a person on the ground in each parish. One team member, Tanya Sullivan-McGee, is a middle school coordinator for her parish, and has particular success convincing the teachers of her parish to attend.

MAKE IT EASY TO ATTEND.

The location of Nakamaye's group meetings is ever-changing, alternately held at various schools throughout the district. He said that teachers from the particular school where the meeting is being hosted are often well represented at each meeting. In his opinion, this is due to the ease of attending for them.

Similarly, Rodin said that her group's consistent location in a university building immediately adjacent to a well-lit parking garage that is free in the evenings at the time of the meetings is an invaluable factor in her group's attendance.

Meanwhile, Covington adds that making it easy to attend is not only about location, mentioning that she also brings notepaper and pencils for the teachers to take notes, so they don't have to remember to bring anything.

MAKE IT WORTHWHILE.

However, the most important factor for a Circle's success is to make the meetings a valuable asset to the teachers in attendance.


"Teachers will only come if it is worth their while to come," Nakamaye said. "There are a variety of things you can try, but it all comes down to that."

Rodin's group, for example, offers Continuing Education Credits and Gifted and Talented Credits to their participants. "Teachers in Texas often pay up to \$100 for events that award these credits," Rodin said. "It didn't cost anything but time to get our MTC certified. It's just

paperwork, but it makes attending a great value for our teachers."

Rodin's group also consistently orders high-quality catered food from the best local restaurants, rather than having pizza at every meeting like many teachers' groups often have. "It costs a little more, but the teachers really deserve it and they really enjoy it," Rodin said. "And we always make sure to have really superb desserts for them as well."

Covington echoes the importance of treating the teachers with the highest respect and the best of everything your Circle can afford. "It's really important to treat your teachers well, like the professionals they are," Covington said. "They're already so overworked and underpaid. The best thing to do to boost attendance is to make the meeting something exciting, enticing, something that people actually want to attend."

In essence, build it well, and, indeed, they will come. 



PARTIAL CONCLUSION PROBLEM POSING STORY/HISTORY THE “GOOD PRO

“Where do you get your ideas?”

is a question that every writer encounters all too often, and writers of problems for Math Teachers' Circles are no exception. To paraphrase Robert Pirsig's advice about painting in *Zen and the Art of Motorcycle Maintenance*, “You want to know how to write the perfect problem? It's easy. Make yourself perfect and then just write naturally.” That's perhaps the most honest—and most useless—possible advice on the topic. It does point in some productive directions, though. What you want to do is not so much learn how to create good problems as to become the kind of person who notices good problems all around you, and to immerse yourself in a culture where you live in a higher density of such problems.

How do you learn to notice those suitable problems when they fall into your lap? For one thing, you need to cultivate the habit of playing with mathematics when you come across it. What are different ways you could look at it? Could you explain some aspects of it to a five-year-old? Can you reach important insights by doing experiments a middle school student could understand? Can you represent the idea in a different way? For instance, the geometric and number-theoretic ideas behind the game of SET were around for a very long time, but the playfulness encouraged by the game and the novel way of representing those ideas led to a lot of great MTC problems^[1] and, ultimately, completely novel mathematics.

While we're looking for problems, we need to keep in mind that we're not hunting down exercises. As Paul Zeitz tells us, “Exercises may be hard or easy, but they are never puzzling” – we are supposed to know already how to approach them. A problem presents us with a novel challenge, where we're not sure what tools we need. By this definition, solving a Sudoku is generally an exercise; it's only the times when we're stuck and need to find a new approach, or a new way to put together old approaches, that qualify as problems. Of course, a simple exercise for an expert might be quite a challenging problem for a beginning solver who doesn't have the same library of techniques and strategies!

That novel challenge means that an intriguing problem feels **perplexing**. Good mathematics should be a tool for resolving perplexity! It's part of human nature to want to solve a puzzle. On the other hand, a puzzle can easily be too easy, so it's just a time waster, or too hard, so that it feels frustrating. So, an important part of the session leader's job is to put together the problem in such a way that it's **tuned to the audience**. Even more importantly, the problem should have plenty of **easy entry points**, so that people at almost any level can get started, with some early examples that contribute to understanding of the depth that will come later. But the learning doesn't come without some amount of challenge and discomfort. “Stuckness shouldn't be avoided,” as Pirsig

CLASSROOM CONNECTION CLEAR & AMBIGUOUS EASY ENTRY POINTS BLEM” PROBLEM by Joshua Zucker

advises. “It’s the psychic predecessor of all real understanding.” On the way to that stuckness, though, the introduction needs to be **concise and accessible**, to get everyone involved.

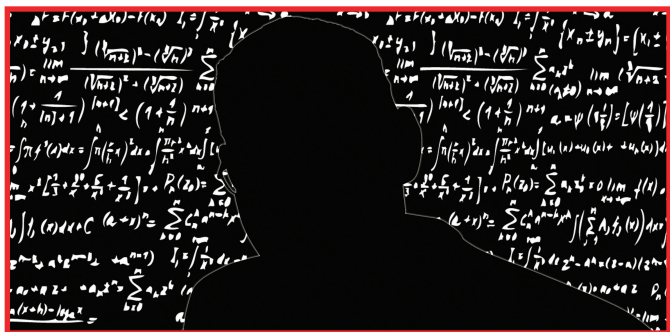
In addition, most in the MTC community agree that good problems should be **both clear and ambiguous**, which seems like about as blatant a contradiction as one could have. Perhaps what we mean is that the communication should be clear but the question might not be. We shouldn’t be afraid to ask non-mathematical open-ended questions like “What happens?” or “What can you see here?” or to create a problem situation where participants then produce their own questions. The art of **problem posing** can be just as much a part of our sessions as the art and craft of problem solving. A good, deep problem leads to bigger problems and generates more questions of its own. So, in a further contradiction, our session might end with only a **partial conclusion**, where we feel satisfied about something having been resolved but perhaps with many further questions remaining to be answered.

The introduction of a perplexing problem can feel almost like a code you need to break^[2] or the beginning of a good mystery novel. In a mystery novel, though, the clues are deliberately hidden and the detective may put them together in a way that seems almost magical. In mathematics, we want our participants to learn the secrets of our detective magic!

Thus, a good problem needs to **illuminate strategies** and techniques so that solvers come away with new tools that they can use to solve future problems. Moreover, they gain an appreciation for novel ways to combine those tools.

We can **connect** disparate problems by means of strategies, and conversely we can connect different areas of mathematics by means of a problem. Sometimes problems that are on the surface quite unrelated^[3] turn out to have some deeper idea connecting them. As mentioned above, the game of SET connects geometry, number theory, combinatorics, linear algebra, and more. Many problems have both numerical and geometric interpretations, where problems on the grid^[4] turn out to lead to deep number-theoretic ideas, or where a familiar multiplication table has a geometric interpretation^[5] that leads to deeper understanding. **Multiple representations** are one way of generating connections and making new discoveries, as well as giving new insight to taken-for-granted fundamentals like place value^[6,7].

These connections and multiple representations often lead to one of the best ways to make a session engaging: **surprise!** The best kinds of surprises are the sudden emergence of a pattern when work on a problem is organized a certain way or the sudden “obviousness” of a difficult fact when it’s looked at from a new perspective. Untie ropes using arithmetic of fractions?^[8] Do number theory and triangle geometry



by pouring water?^[9] After some MTC experience, these kinds of surprises turn out to be, well, surprisingly common. So, cultivate a sensitivity to this kind of solution. Peter Winkler's books, as well as Martin Gardner's and Ian Stewart's, are excellent resources for problems with a great surprise.

To keep people engaged in working on the problems, it also helps to have a few key planned **landmarks** along the way. These can be "A-ha!" moments of breakthroughs, conclusions about one aspect of a problem, "Oops!" moments where something that everyone was assuming turns out not to be true, or "Now what if?" moments where variations on a theme can be explored. These landmarks can also be summaries of key discoveries. These key points help make it possible for participants to leave satisfied with their different levels of understanding.

Perhaps the question that most distinguishes the approach of various Circles in choosing problems is the extent to which our sessions should have a **classroom connection**. This can take many forms. For example, the topic can be directly related to middle school content standards, such as fractions^[10], even if our exploration of it goes far beyond the standards. Or, we can choose a topic that embeds skills from the standards in a very non-standard way, like Rational Tangles^[8]. There are advantages to this: teachers may be able to use a form of the material in their classroom as part of their normal lessons. But there are also disadvantages: teachers with their "teacher hat" on may end up feeling more like they're at work instead of playing with mathematics, and they may think more about how to communicate the results to their students instead of generating more results on their own. Whether or not there is a direct connection to content standards, good MTC problems should

always have **mathematical practices** as a primary focus.

For many writers and solvers, one of the most appealing features of great problems is a compelling **story or history**. This makes the problems more memorable, and makes us all more a part of the culture of problem solving with its shared folklore. It also makes it more likely that participants will "leave the session **humming the problems**," as Ravi Vakil describes the goal of an author. There's no more satisfying compliment than to find that the participants have continued working on the problem long after the end of the session! ☐

Links and Resources

1. [The Game of SET](#), B. Conrey and B. Donaldson.
2. [Codes, Ciphers and Secret Messages](#), M. Beck.
3. [The Power of Wishful Thinking](#), A. Rodin, and [Two is Better Than Infinity](#), P. Zeitz.
4. [Grid Luck](#), T. Shubin.
5. [Multicolored Multiplication Table](#), D. Millar.
6. [Coins in Twoland](#), J. Zucker.
7. [Exploding Dots](#), J. Tanton.
8. [Conway's Rational Tangles](#), T. Davis.
9. [After the Jug Band](#), E. Johnston, and [The Three Jugs Problem](#), J. Zucker.
10. [Fractions, Decimals, Ratios, Rates, Percents, Proportions](#), T. Shubin, and [Fractions and Decimals](#), T. Davis.

For links to these resources and more, visit us online at <http://mathteacherscircle.org/resources/sessionmaterials.html>.

Investigations in the Classroom

Study Finds MTC Participation Leads to More Inquiry-Based Teaching

David H. Khaliqi, Peter D. Marle, and Lisa L. Decker of the University of Colorado Colorado Springs have recently reported some preliminary findings from an ongoing study of teacher participants in the Pikes Peak Math Teachers' Circle. The study examines MTC participants' teaching self-efficacy, pedagogical content knowledge, and use of reform-oriented (i.e., inquiry-based and investigative) teaching practices.

Currently, two groups of teachers have completed the year-long study. Results to date indicate that after one year of MTC participation:

- Surveys of teacher self-efficacy showed increased feelings of being pedagogically prepared, more attunement to investigative culture and practices, and increased personal math teaching self-efficacy.
- Teachers reported increased use of reform-based teaching practices during interviews.

- Consistent with teachers' own reports, classroom observations provided evidence of significant increases in the use of inquiry-based teaching practices. In addition, the observations showed significant increases in teachers' pedagogical content knowledge.

Khaliqi, Marle, and Decker presented their results at the School Science and Mathematics Association National Convention in Birmingham, Alabama, in November. They will also present this research, combined with data from a third cohort of teachers, during the Research Pre-session for the National Council of Teachers of Mathematics Annual Conference in Denver, Colorado, in April 2013.

For more information about this research, please view the slides on our [website](#) or contact Lisa Decker at ldecker@uccs.edu. 

got news? events?

Keep the rest of the MTC Network up-to-date with events and news in your own Circle. It's as simple as submitting news items and notice of your upcoming events to

circles@aimath.org

News may be published on our website or in a future issue of MTCircular.

Events will be included on our online calendar at

<http://www.mathteacherscircle.org/calendar.html>

Learning to Lead

Fawn Nguyen Offers Her Perspective on AIM's "How to Run a MTC" Workshop

I have a tendency to get myself into situations that I wasn't expecting. I never know what I'm getting myself into, but it can lead to great things. All I wanted was to find a Math Teachers' Circle workshop to attend, and when there wasn't one in my area, my ambitious evil twin set off to start one.

Eventually this led me to the AIM offices in Palo Alto for a week-long workshop last summer on "How to Run a Math Teachers' Circle."

I had asked Erin Hanley, my next-door math colleague, to join me. She and I fulfilled the requirement for "two middle school math teachers" — we just needed to find "two mathematicians and one administrator or organizer" to make a team of five.

I contacted Brianna Donaldson, AIM's Director of Special Projects, to help us round out the team. As it turns out, she had recently been contacted by Nate Carlson, who had previously helped start an MTC in Arizona as a graduate student and now, as a professor at California Lutheran, had been working to start one in Thousand Oaks together with his colleague Hala King. Serendipitously, they had the professor side of the group, seeking out teachers, while at the same time we had the teacher side of the group, seeking out professors. We still needed a fifth member, so I asked Melissa Diaz, another math colleague, and she graciously got on board. Our group was complete.



The Thousand Oaks MTC leadership team poses for a photo on the last day of the workshop. From left, Nate Carlson, Fawn Nguyen, Melissa Diaz, Erin Hanley, and Hala King.

Our team received full funding to participate in the workshop, which was full of fascinating math problems and great fellowship. After a two-hour lunch each day, we would work within our group on plans for logistics and fundraising for our own MTCs. And then, if we wished to stay, "Happy Hour" greeted us to close each day. I never drank so much wine in one week! Here are the day-by-day highlights of the rest of the workshop.

Monday

I meet the other two members of our team, Nate and Hala, for the first time. There are six teams, from Texas, Kansas, New York, and California.

Josh Zucker appears first on the agenda with an "Introduction to Problem Solving." I've always wanted to meet Josh — his name appears on most of the cool math problems that I encounter at the Julia Robinson Mathematics Festival in Los Angeles. Turns out he's the director. He facilitates this classic problem:

The numbers 1 through 100 are written on the board. You choose any two numbers x and y and erase them, writing $x + y + xy$ in their place. You continue to do this until one number remains. What are the possible values for that remaining number?

(I want to present just the problems here so that you — the thinker, the mathematician, the teacher, the problem solver — get to struggle with the problem and construct meaning for yourself. If there's a problem that interests you, you can find links to in-depth handouts in the "Links and Resources" box at right.)

Solving this particular problem makes me crazy happy, because it shows me our team is already working great together. I will try to see rectangles in almost every math problem, and I get stuck solving this problem with a rectangle drawn on my paper. But Erin sees something from it and brings the problem home, and Nate concurs with her answer. We are clicking marvelously.

Tuesday

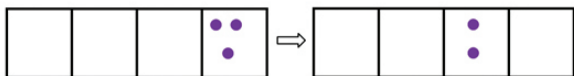
Tatiana Shubin, a math professor at San Jose State University, presents “Grid Power.” I’ve always required my students to use quadrille-ruled composition books to take notes, but after hearing Tatiana speak, I want my kiddos to do ALL their math work on grid paper!

Tatiana gives us a delightful and deceptively simple problem: How many squares are in a 7×7 square?

Later in the day, Paul Zeitz, a math professor at the University of San Francisco currently on sabbatical, introduces us to “Mathematical Games” like Takeaway and Puppies and Kittens, a.k.a. Wythoff’s Nim. I’ve ALWAYS wanted to buy the book *The Art and Craft of Problem Solving*, and lo and behold, Paul is the author! But Paul doesn’t once mention his book (I made the connection after the workshop); instead he recommends James Tanton’s *Solve This*. I need these books.

Wednesday

Diana White, a math professor at the University of Colorado Denver, facilitates us through the “Exploding Dots” problem. Say you have a machine that holds ping-pong balls. If you put three balls in the far right slot, they’ll explode and two balls will move one space left into the next slot. Like this:



This happens with any set of three balls in one slot, therefore, the explosion continues until there are fewer than three balls per slot. Thus, starting with 9 ping-pong balls, the result looks like this:



Later in the day, retired engineer Tom Davis walks us through “Conway’s Rational Tangles,” a wonderful activity that kids and teachers will absolutely love to get their hands on, literally.

Thursday

Paul Zeitz is back this morning with “How to Gamble, If You Must.” We play a few dice games, then we work on various lottery scenarios.

Friday

There is no math activity on our last day. Each team has 10 minutes to present the plans for their own future Math Teachers’ Circle. We each get a T-shirt and our team pictures taken, and we bid farewell at noon. I’m so grateful to everyone who made this workshop possible. And I love my team! ☑

Fawn Nguyen is a math teacher at Mesa Union Junior High School in Somis, California, and a founder of the Thousand Oaks MTC. This article was adapted from a blog post by Nguyen.

Links and Resources

[Finding Ways to Nguyen Students Over](#), Fawn Nguyen’s blog.

[Thousand Oaks MTC home page](#)

[Julia Robinson Mathematics Festival](#)

[The 1-to-100 Problem](#), J. Zucker and T. Davis.

[Grid Luck](#), T. Shubin.

[The Art and Craft of Problem Solving](#), P. Zeitz.

[Solve This](#), J. Tanton.

[Mathematical Games](#), P. Zeitz.


[Exploding Dots](#), J. Tanton.

[Conway’s Rational Tangles](#), T. Davis.

[How to Gamble, If You Must](#), P. Zeitz.

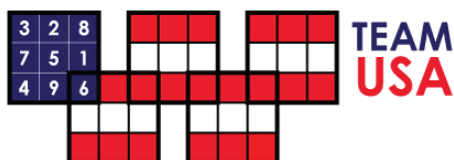
For links to these resources and more, visit us online at <http://mathteacherscircle.org/resources/sessionmaterials.html>.

MTCs Recommended as Professional Development for In-Service Teachers


The Conference Board of the Mathematical Sciences (CBMS) recently released the Mathematical Education of Teachers II (MET2) document, which contains recommendations on pre-service teacher education as well as continuing professional development for in-service teachers. Math Teachers' Circles are highlighted as a recommended form of professional development for middle and high school teachers. According to MET2, "A substantial benefit of [Math Teachers' Circles] is that they address the isolation of both teachers and practicing mathematicians: they establish communities of mathematical practice in which teachers and mathematicians can learn about each others' profession, culture, and work." The full document is available on the CBMS website, <http://www.cbmsweb.org/>. 




AIM/MSRI's Joshua Zucker Competes with Team USA at World Sudoku Championship



Joshua Zucker of AIM and MSRI competed as a part of the U.S. Sudoku Team at the 7th World Sudoku Championship in Kraljevica, Croatia, in October. The team placed tenth in the overall competition ranking at the event, which is held in conjunction with the World Puzzle Championship held by the World Puzzle Federation each year. Zucker, who is also a

regular editorial contributor to MTCircular, qualified for a spot on the team by earing a high score on the qualifying test, along with Jonathan Rivet and Jason Zuffranieri, who joined Wei-Hwa Huang to form the four-person Sudoku Team. The U.S. Puzzle Team also fared well at the event, earning the third spot in the overall competition while USA team members Thomas Snyder and Palmer Mebane took second and third place respectively in the individual competition. For more information on the event, visit <http://www.wscwpc2012.org/>. 

AIM Thanks Math for America for Generous \$24,000 Donation to Fund MTC Seed Grants

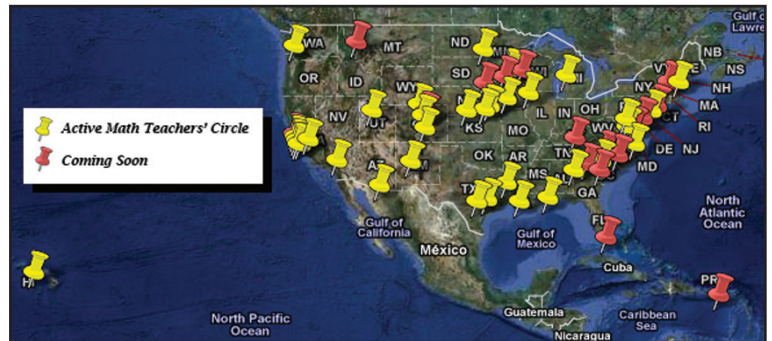
Math for America recently donated \$24,000 to the American Institute of Mathematics to fund seed grants to start new MTCs. This generous donation will help these groups fund initial meetings and attract further support. The funds will be preferentially awarded to teams who attended a 2012 "How to Run a Math Teachers' Circle" workshop. Details on how to apply for the funding will be available in early 2013. 



A Warm Welcome to Our Newest MTCs

The nationwide network of Math Teachers' Circles is growing. Each of these groups sent a leadership team of about five people to one of our Summer 2012 workshops on "How to Run a Math Teachers' Circle." It is expected that they will each start their own MTC by Summer 2013. These new MTCs will join approximately 44 active MTCs and several others in the planning stages across the United States and its territories. A warm and hearty welcome to the Network's newest members:

- Stockton, California: San Joaquin County MTC
- Thousand Oaks, California: Thousand Oaks MTC (<http://www.mathteacherscircleto.org/>)
- Olathe, Kansas: Kansas City Area MTC
- Frederick, Maryland
- Jackson, Mississippi: Capital City Circle (<http://www.capitalcitycircle.com/>)
- Annandale-on-Hudson, New York
- Rochester, New York
- Stony Brook, New York
- Athens, Ohio: Southeast Ohio MTC (<http://seomtc.weebly.com/>)
- Cincinnati, Ohio: Cincy Math Circle (<http://cincymathcircle.wikispaces.com/>)
- Nacogdoches, Texas
- Seguin, Texas ☑



Visit our website at <http://www.mathteacherscircle.org/> for an interactive version of this map.

Charlotte MTC Leader Harold Reiter Receives UNC Board of Governors Public Service Award



Dr. Harold B. Reiter, a founding member of the national MTC Network's leadership team, was awarded the University of North Carolina System's Award for Excellence in Public Service in October. As MTCircular reported previously, each of the University of North Carolina's 17 campuses nominates one faculty member for consideration for the award. Reiter was nominated by UNC Charlotte in recognition of his sustained public service, including his involvement in the MTC Network, and went on to win the award, which carries a \$7,500 cash prize. The award was established in 2007 to encourage, identify, recognize, and reward distinguished public service and outreach by faculty across the University. Reiter has been involved with the MTC Network since its inception and has played an important role in the development of the successful national initiative. He attended the very first MTC workshop in 2006, and began efforts that same year to found a MTC for middle school teachers in the Charlotte area. The Charlotte Teachers' Circle was the first MTC to be formed outside the San Francisco Bay Area, reflecting Harold's commitment to bringing this innovation to the teachers of North Carolina. ☑

MTC Advisory Board Member Jane Porath Elected to NCTM Board of Directors



MTC Advisory Board Member Jane Porath has been elected to an At-Large position on the Board of Directors for the National Council of Teachers of Mathematics. The Board sets the direction, establishes policy and oversees the activities of the NCTM. Porath, a math teacher and department chair at Traverse City East Middle School in Michigan, has been involved in NCTM activities since 1998 and has served on the MTC Advisory Board since 2010. Porath was elected in a general vote of NCTM members to serve a three-year term on the Board, beginning at the conclusion of the NCTM 2013 Annual Meeting and Exposition in April. “As classroom educators, we are experiencing difficult times in this era of testing and budget shortfalls,” Porath said in a statement. “It is imperative that we find our voices and stand up for mathematics education. One of the best ways to do this is to continue to be the highly educated professionals that we are. Involvement and service to organizations such as NCTM help us maintain our dignity and remind us of why we chose this career: to educate our youth and make a positive difference in their lives.” Read more at



<http://www.nctm.org/>. 

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Photos, clockwise from top left: Martyn E. Jones, Reno Rodriguez, Martyn E. Jones, and Katie Bianchini, stock.xchng

Problem Circle by Joshua Zucker

Simple, or Impossible?

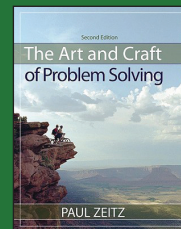
$$x^x \cdot y^y = z^z$$

I'm surprised that it took me this long in life for anyone to ask me to find integer solutions to this equation. The question just seems so natural. It seems like something that a bored kid in algebra class would invent, or that Fermat would doodle about in a margin. Thanks to Alon Amit, my wait is over.

Of course, there are some easy integer solutions to find, but if we require that all the variables are greater than 1, it gets a lot more difficult. For me, this was one of those magical problems that balances on the edge: are there solutions, or not? I would begin by trying to find a solution, then get stuck, try to turn the stuckness into a proof that there is no solution, find a flaw in the proof attempt, and go back to looking for a solution again. ☐

WIN A FREE BOOK!

Got an answer to this issue's Problem Circle? Send it in and you could win! MTCircular is offering a copy of any of the books listed at <http://mathteacherscircle.org/resources/general.html> or <http://mathteacherscircle.org/resources/sessionmaterials.html> to the best solution or partial solution. Send your answers to problemcircle@aimath.org for consideration.



Mary Fay-Zenk and Richard Grassl submitted a partial solution to last issue's problem on Number Sum Triangles. They found a few of the hundreds of patterns that can be uncovered here. They point out that each row is symmetric; that starting from the center, which is even after the first row, on each side there's a repeating pattern of two odds and an even; and that each row sum is one more than a power of 3. (For this last reason, among others, people often leave off the second 1 when writing this sequence.)

They also observe some of the more amazing patterns. With the way the sequence was presented here, it's more natural to center the rows. But if you left-align them, each column is an arithmetic sequence! Even more astonishing, the common differences of those arithmetic sequences are again a copy of the original sequence.

Another way to align the numbers is to space them evenly, so that the 1's on the ends form two columns, as shown at right. Now you can see that once a number appears, it will continue in its column forever. Also, you can see that, for example, the 4's produced in the 4th row head the columns of all 4's that will ever occur, because in future rows adjacent pairs will only ever have sums larger than 4.. For a further discussion of this sequence, visit our website at <http://mathteacherscircle.org/newsletter/problemcircle.html>. ☐

1				1				
1		2		1				
1	3	2	3	1				
1	4	3	5	2	5	3	4	1

Ron Graham's Sequence

by Susan Holtzapple and Joshua Zucker

Ron Graham invented a delightful sequence of integers with several amazing properties you can discover for yourself, or if you want to spoil your fun, by reading through this article to the end. In a recent Math Teachers' Circle session, we introduced the idea by taking a non-example and gradually refining it until it fit the rules for constructing Graham's sequence. Showing both non-examples and examples is always a useful way to introduce a new definition. So we began with this list of tasks, with sample answers shown here:

1. Make a sequence of positive integers (e.g., 1, 1, 2, 3, 5, 8, ...)
2. That is also finite (e.g., 1, 2, 3, 4)
3. That also begins with 6 (e.g., 6, 6, 6)
4. That is increasing (e.g., 6, 7, 8)
5. So that the product of all its terms is a perfect square (e.g., 6, 600)

Given several possible finite increasing sequences beginning with 6 whose product is a perfect square, let's choose the best one. What makes it best? There are a lot of possible ideas, and we can spend some time hunting for record-breakers in each category and for proofs that we've found the ultimate record-setter. For instance, we might look at fewest terms (6, 600 for example), smallest perfect square product (6, 24), smallest last number (6, 8, 12), or any other way that the sequences could compete for the title of "best."

Let's focus on the smallest last number. Of all the sequences starting with 6 that satisfy our rules 1-5, the one with the smallest last number is (6, 8, 12). This defines 12 as the 6th term of Graham's sequence. To find the 2nd term, generate sequences starting with 2 instead of with 6 (i.e., Rule 3 will read, "That also begins with 2" and the other rules will remain the same). You'll find that the sequence with the smallest last number is (2, 3, 6), so 6 is the 2nd term of Graham's sequence. Try working out the first 10 terms of the

sequence before looking at the figure at the bottom of this page. The bottom row of the figure contains the first 10 terms of the sequence, and the columns contain the finite sequences that generate the terms. In a MTC session, the participants might work collaboratively to generate the first 30 or so terms of the sequence.

Somewhere along the way, people will discover the utility of a few "magic" numbers like 8 and 18. Why do they seem to come up so often? Well, they're like inserting an extra 2, because apart from a factor of 2, they won't change the "perfect squareness" of the product. This will lead to the most important idea behind this sequence: we can focus on the prime factorization and simplify our search. A product is a perfect square if, and only if, each prime shows up an even number of times in total. For instance, in 10, 12, 15, 18 we have $2 \cdot 5$, $2 \cdot 2 \cdot 3$, $3 \cdot 5$, and $2 \cdot 3 \cdot 3$, for a total of 4 twos, 4 threes, and 2 fives. We can see that the product is a perfect square without even knowing what the product is!

After some work constructing the sequence, we can explore in a few different directions. We might notice some patterns in the sequence and try to prove them! One that people often seem to notice is that for any prime p , the p^{th} term of the sequence seems to be $2p$, except maybe for a few exceptions right at the beginning. Is this always true?

Once you understand what's going on with the p^{th} term of the sequence being $2p$ as long as p is big enough, is there something similar about the $2p^{\text{th}}$ term

1	2	3	4	5	6	7	8	9	10
	3	6		8	8	8	10		12
							12		15
1	6	8	4	10	12	14	15	9	18

being $3p$? Does this generalize?

Another pattern people often notice is that the sequence seems to skip over the primes. They usually explain the proof that if one of the sequences ended with a prime, then the product of the terms would be divisible by the prime but not its square, so it couldn't be a perfect square. This leads to the difficult question of whether all the non-primes eventually appear.

At some point along the way, you probably had two people suggesting different sequences for the same number. For instance, what if you have the sequence 10, 18, 20 and someone else has the sequence 10, 12, 15, 18? After verifying that both products are perfect squares, you can see that the 10th term of Graham's sequence is not 20: it is at most 18, but by focusing first on the fives and then the threes and then the twos you can establish that 18 can't be beat.

But instead of leaving it at that, there's another discovery to be made: given any two sequences whose product is a perfect square, if you combine all the terms, the product is still a square, since $a^2b^2 = (ab)^2$. So we know that 10, 18, 20, 10, 12, 15, 18 has a perfect square product. We sort it so that it is increasing: 10, 10, 12, 15, 18, 18, 20. Now we delete the repeated factors, since we can divide out perfect squares, and we have 12, 15, 20 as a perfect square product sequence. So, combining two sequences with the same starting number yields a new sequence with a bigger starting number and the same largest number as before.

What good is that process? Well, for example, suppose we wanted to prove that there is some term of the sequence equal to 120. Looking at the prime factorization of 120, we can tell that we need a 2, a 3, and a 5. So, we know that the sequence 2, 3, 5, 120 has a perfect square product. But the second term of the sequence is not 120, of course! It's 6, because of 2, 3, 6. Now we can combine these two sequences and we have 5, 6, 120. In

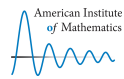
other words, because 120 wasn't the second number of the sequence, we could create a new sequence, still ending with 120, but with a larger starting number. So now, either 120 is the fifth term of the sequence, or we have a better sequence starting with 5. Since we have 5, 8, 10, we again combine to get 6, 8, 10, 120. The only way this process can stop is if we have a starting term where there's no smaller possible ending term than 120. And when we get there, we've figured out where in Graham's sequence 120 appears!

You might enjoy working out the details there. To us, the location of 120 in the sequence turns out to be quite a surprise, with prime factors we wouldn't have suspected could possibly be involved in 120.

There's one more thing to observe, too. You might find that no number appears twice in Graham's sequence. For instance, if you have two proposed sequences with the same ending number, like 10, 18, 20 and 12, 15, 20, you can combine them in the same way to get 10, 12, 15, 18, 20, 20 and then 10, 12, 15, 18. This shows that the smaller of the two proposed starting numbers must have a smaller ending number!

So, what have we proved? We know that no prime numbers appear, and every non-prime number appears, and no number appears twice. That is, we have a list of all the composite numbers in some order! Not only that, but we can figure out the number that appears in any particular spot in the list without having to compute all the previous numbers. We doubt it's possible to do something similar with the primes! If it is, please let us know. ☐

Susan Holtzapple, a middle school mathematics teacher in Cupertino, CA, and Joshua Zucker, a freelance math teacher who works extensively with MTCs and Math Circles for students, are members of the AIM MTC, where they led a session on this topic in September.



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