# MTCircular

## TAKING ROOT

### NURTURING A GROWING MATH TEACHERS' CIRCLE NETWORK
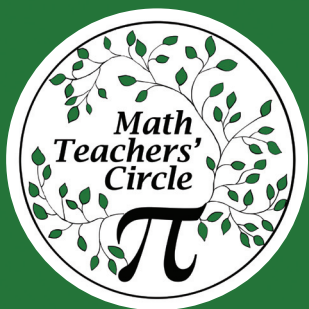
+

Math Teachers' Circle

# Growing From the Roots Up

Dear Math Teachers' Circle Network,

Over the past eight years, Math Teachers' Circles have grown slowly but surely, until our network now encompasses 71 Circles in 36 states. Yet we feel that MTCs could play a larger role in U.S. mathematical culture. Realizing that the "How to Run a Math Teachers' Circle" workshops would only take us so far, we decided to develop a new approach to expanding our network. I'd like to thank our Leadership Team for all their contributions to this process and also to acknowledge the tremendously helpful guidance of our Advisory Board, in particular Uri Treisman.

In designing our scale-up efforts, we have focused on two guiding principles:

- Preserving the character of MTCs as ongoing professional communities of mathematics teachers and professors focused on doing mathematics together

- Nurturing our network from the roots up by sustaining existing MTCS and building on their outstanding achievements as part of the process of beginning new MTCs

The resulting process is intended to deepen our roots through regional networks that will grow from a system of Sister Circles and Circle Mentors. You can learn more in the "Taking Root" article from this issue, and by exploring our website at **http://www.mathteacherscircle.org**.
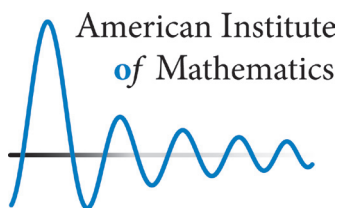
Bob Klein and Steve Phelps's featured session on "Liar's Bingo" is a great illustration of collaboration across MTCs in Ohio, where there is now a statewide network of MTCs supported by a grant from the Ohio Board of Regents.

I'd also like to draw your attention to a new feature, "From the Circle to the Classroom," in which teachers share classroom-tested middle school lessons connected with their MTC experiences. I hope you enjoy Corinne Cristiani's delightful lesson on compositions and that some of you try it out with your own students!

Happy problem solving!

*Brianna Donaldson*

Brianna Donaldson, Director of Special Projects

American Institute *of* Mathematics

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Cover Photo: iStockPhoto

# TAKING ROOT

**M**ath Teachers' Circles are about mathematics. They bridge the divide between teachers and professors by developing a common language grounded in the power and beauty of mathematics. They support a K-20 professional mathematical community with the shared responsibility of educating the next generation and the ultimate goal of transforming our culture's view of mathematics.

Teachers are on the front lines of the mathematical community. Our vision is for every teacher in the United States to have access to a Math Teachers' Circle. We have set an initial goal of 300 Math Teachers' Circles focused on middle school teachers by 2019. We will branch out to new locations while also deepening the roots of our current Circles in ways that will contribute to their long-term sustainability and the sustainability of the MTC Network.

Here, we highlight some of the biggest changes to the Math Teachers' Circle Network for new and existing Circles. Please also visit our website at **http://www.mathteacherscircle.org** to learn more.

## Starting a New Math Teachers' Circle

From 2007 to 2014, AIM organized annual workshops on "How to Run a Math Teachers' Circle," with support from the Mathematical Association of America, the National Security Agency, and the National Science Foundation. These workshops evolved significantly over time and developed into a highly successful means of beginning MTCs, with approximately 90 percent of teams who participated in the last three years of workshops forming a Math Teachers' Circle.

With space for only 12 teams each summer, however, the workshops are not able to meet the increasing
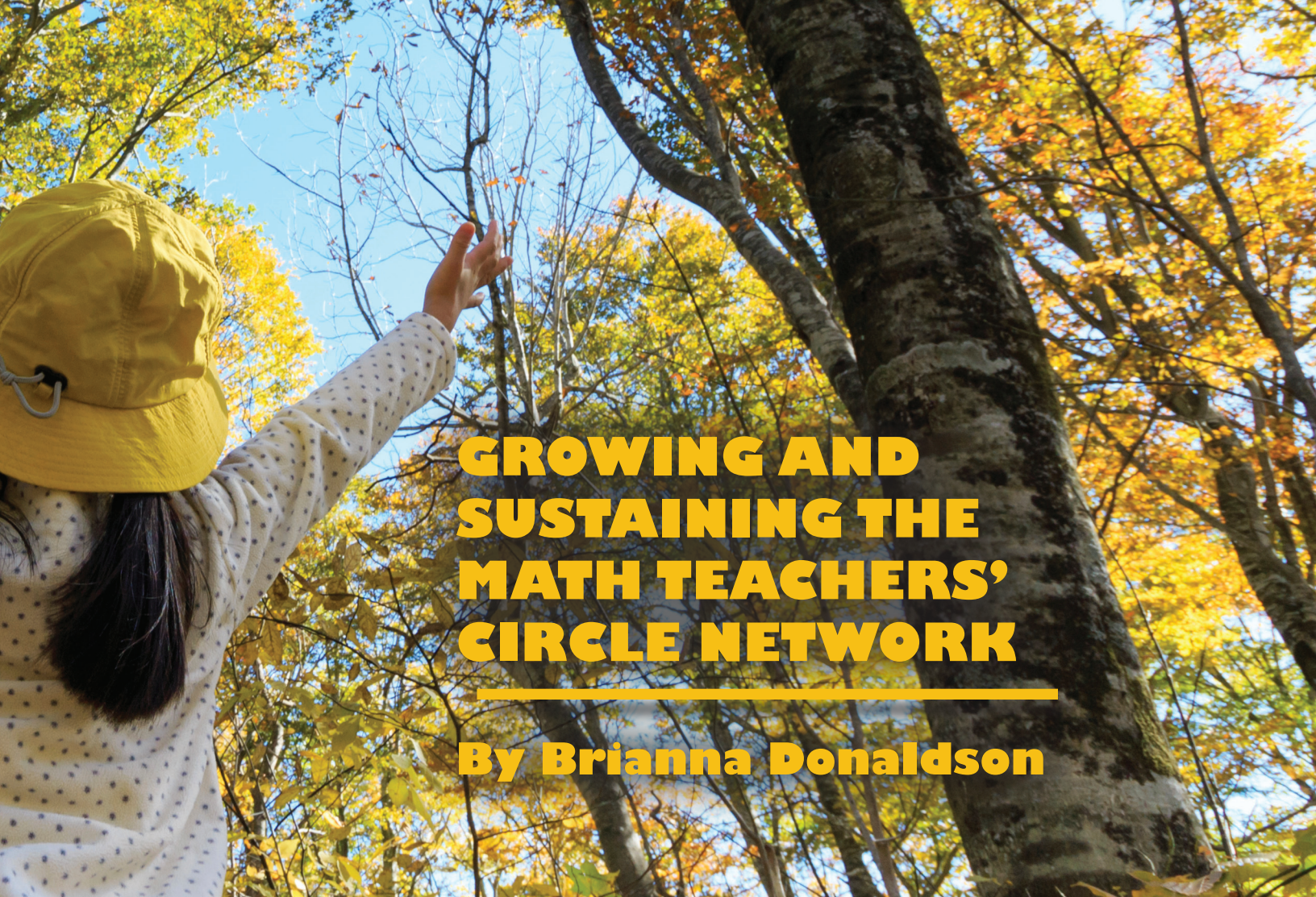
# GROWING AND SUSTAINING THE MATH TEACHERS' CIRCLE NETWORK

## By Brianna Donaldson

demand for creating new Math Teachers' Circles. Furthermore, the centralized workshop model does not make the most of the distributed expertise of Math Teachers' Circle leaders. Our new hybrid model for starting new Math Teachers' Circles integrates centralized Web and staff resources with regional support from Sister Circles and a mentor network of expert leaders around the country.

## Application Process

To begin your Math Teachers' Circle, you will need:

- A leadership team of two middle school teachers and two college or university mathematics faculty members

- A host institution committed to supporting your Math Teachers' Circle

Once you have a leadership team and a host institution, you can apply for support from the Math Teachers' Circle Network. If your MTC is selected for support, we will help you plan the logistics of your MTC, sponsor your leadership team to visit a nearby "Sister Circle," sponsor a Circle Mentor to visit your MTC, and provide a small amount of start-up funding to help with the expenses for your initial summer workshop and first year of meetings. We will also connect you with relevant resources, such as our organizer toolkits and videos.

## Online Resources

### Organizer Toolkits

The organizer toolkits consist of carefully selected resources organized around topics such as "Beginning Your Circle" and "Sustaining Your

# Hosting a Math Teachers' Circle

## Why should my math department host a Circle?

- Hosting a Math Teachers' Circle demonstrates that your department is involved with and committed to the local education community.

- You will maintain ongoing ties with local teachers who are department alumni or who have completed other programs through your department. Not only will you enjoy continuing to work with them mathematically, but they also may be able to help you with pre-service teacher placement or with guiding talented undergraduate students to your department.

- Faculty can share their enjoyment of mathematics with other professionals, while at the same time demonstrating a concrete commitment to the broader impacts of their mathematical work.

- Graduate students can develop their teaching and outreach credentials, positioning them well in a highly competitive job market.

- Pre-service teachers can observe a professional learning community in action and make connections with local practicing teachers, enriching their educational experience and potentially increasing their local employment opportunities.

An article describing the benefits of Math Teachers' Circles for mathematicians appeared in the December 2014 issue of the Notices of the American Mathematical Society. Read it online at **http://www.ams.org/notices/201411/rnoti-p1335.pdf**.

## What commitment is involved?

At a minimum, your department would commit to providing:

- Space for the MTC's meetings, which typically take place 6 to 8 times during each school year

- Faculty member facilitators to lead mathematical sessions for the MTC

Here are some other ways a host institution can support the work of a MTC:

- Host the MTC's website

- Donate copies and other supplies

- Provide dinner/refreshments

- Provide staff support or a student worker to handle administrative duties for the MTC

Circle: Common Challenges." Additional toolkits will include a guide for session leaders and a toolkit for Circle Mentors. Each toolkit will integrate checklists, documents and templates from our current planning page, articles from previous newsletters, "models that work" examples based on the experience of actual MTCs with different strengths and challenges, and relevant external links.

## Mathematical Resources Database

Development is underway on a mathematical resources database that will enable leaders and participants to interactively browse session leader notes, participant handouts, and classroom materials that have been developed by MTC leaders around the country, and to contribute their own materials. We anticipate this database will be available by early Summer 2015.

## Video Library

We are developing a high-quality video library of mathematics sessions from our "How to Run a Math Teachers' Circle" workshops. The videos are organized into playlists of annotated clips and can be found on our videos page (**http://www.mathteacherscircle.org/resources/videos/**). We anticipate the videos will have a number of different uses, for example, to familiarize new leadership teams and new session leaders with what MTC sessions are like, to give session leaders ideas about how to approach particular sessions, and to provide an opportunity for session leaders and participants to reflect on their own practice through observing the practice of others.

## Community Support

### Sister Circles

Sister Circles are MTCs within driving distance of each other. Whenever possible, new MTCs will be matched with a Sister Circle that their leadership team can visit with the support of the MTC Network. We eventually see regional networks of MTCs growing from Sister Circle connections. These already exist in some areas, for example, the states of Nebraska and Ohio, and the San Francisco Bay Area.

## Circle Mentors

Circle Mentors are experienced MTC leaders who have agreed to help new MTCs get started as well as to consult with nearby Sister Circles who are in need of assistance. Circle Mentors are recognized on our website and receive travel support and a small stipend when they assist with starting up a new MTC.
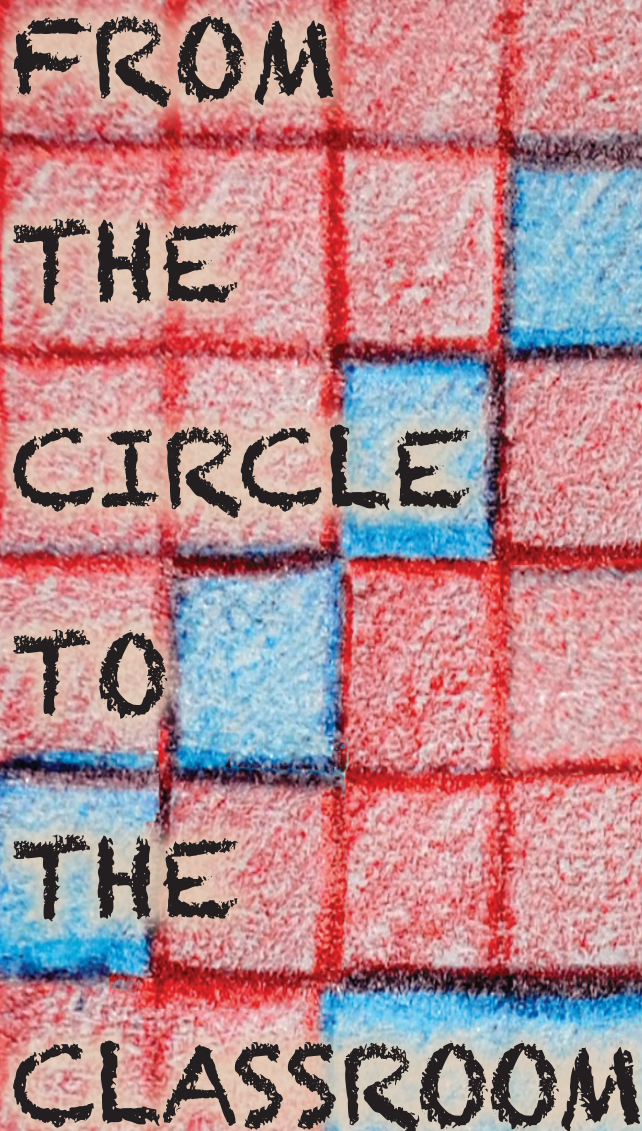
## Additional Consulting

We have always done this upon request, but we will now officially offer consulting services to Circles who need assistance with issues such as:

- Finding a guest session leader
- Identifying possible funding sources
- Letters of support
- Ideas for increasing membership
- Developing a leadership succession plan
- Developing a plan for managing administrative duties for your Circle

Beginning in Spring 2015, we will also hold regular office hours by Google Hangout.

## Looking Ahead

We are excited about strengthening and expanding Math Teachers' Circles and look forward to our work together!

# FROM THE CIRCLE TO THE CLASSROOM

# A MIDDLE SCHOOL LESSON EXPLORING COMPOSITIONS

## BY CORINNE CRISTIANI

*Editor's Note: Last summer, six veteran AIM MTC participants spent a week working with each other, with mentor Mary Fay-Zenk (herself a veteran teacher as well as a co-founder of MTCs), and with local mathematicians, focusing on developing a classroom lesson inspired by their MTC experiences. Several teachers will also co-lead MTC sessions with AIM MTC mathematicians throughout the 2014-2015 school year.*

*The following is an excerpt from a lesson developed by Corinne Cristiani, who teaches mathematics at Crystal Springs Uplands School, Hillsborough, CA. The lesson was inspired by Joshua Zucker's MTC session on compositions (see the Autumn 2012 MTCircular, pp. 18-19, http://issuu.com/mathteacherscircle/docs/mtcircular_autumn_2012). The other veteran teachers, along with their lesson plan topics, are listed in the sidebar, "More to Explore," at far right.*

## INTRODUCTION

Beginning with the deceptively simple idea of compositions, which decompose positive integers into sums of positive integers, students will explore and explain myriad patterns. This lesson is appropriate for middle school, and is both accessible to all students and extendable for advanced students. It provides an opportunity to work with many of the Common Core State Standards for Mathematical Practice, particularly MP7: Look for and make use of structure. No special materials are required, but quad-ruled paper and colored pencils may be helpful.

# OBJECTIVES

Students will employ the mathematical practices of systematic thinking, multiple representations, organizing data, and explaining patterns using both recursion and explicit rules. They will also encounter powers of 2 and Pascal's Triangle. Extensions will expose students to combinations.

# INSTRUCTIONAL PLAN

## INTRODUCING THE PROBLEM

Begin by illustrating a few ways to decompose a particular number. For example, show that:

$$4 = 1 + 2 + 1 = 2 + 1 + 1$$

These two compositions should allow students to see that in this activity, order matters. The rule for writing the compositions of a particular integer is that you may write it as **a sum of one or more positive integers**. This bolded phrase will allow students to decide if a particular composition is valid. Allow students to share other compositions, and decide as a class if they are valid by using this definition. Once students are comfortable with writing compositions, they are ready to investigate this problem on their own.

## INITIAL EXPLORATION

*"How many compositions does the number 10 have?"*

This simple question is a nice entry point for igniting students' curiosity, but there are far too many compositions for a student to fully generate. Given the magnitude of this list, try to limit their initial exploration to 3-5 minutes.

## WHOLE GROUP CHECK-IN #1

**Goal #1: Reinforce *estimation* as a problem-solving tool.**

Ask students to write down their best guess for the number of compositions that 10 has. Ask them to also write a number they are sure that it exceeds, and a number they are sure it is less than. Take a quick poll

## MORE TO EXPLORE

The full text of Corinne Cristiani's lesson, originally entitled, "2 = 1 + 1… and Other Problems Cavemen Could Conquer," can be found at **http://www.mathteacherscircle. org/newsletter**, along with additional teacher notes and solutions. Here are some other lessons developed during the veteran MTC teachers' workshop:

**Gloria Brown Brooks,** *Santa Ana Opportunity School, Hollister, CA* "Platonically Solid"

**Mary Fay-Zenk,** *Miller Middle School, Cupertino, CA (retired)* "Infinity Explorations"

**Tom Freeman,** *San Jose Math Circle, San Jose, CA* "Perspective and Projective Geometry"

**Peter Herreshoff,** *Gunn High School and Terman Middle School, Palo Alto, CA* "Parabola Explorations"

**Susan Holtzapple,** *Cupertino Middle School, Cupertino, CA* The "1-2-3-4 Game" and Variations

**Marc Roth,** *Woodside Learning Center, San Francisco, CA* "Universal Cycle Bead Necklaces"

to determine the highest guess made and the lowest guess made.

### Goal #2: Help students determine the need for *starting with a smaller case.*

If many students estimate that 10 has 100 or more compositions, this may be a good time to lead students toward the strategy of starting with a smaller case in the next part of the investigation. They may then return to their investigations. As students work, encourage those who do not begin with decomposing the number 1 to do so, but only if they have had the opportunity to arive at that strategy on their own first.

## WORK TIME/STUDENT SUPPORT

Allot 15 minutes for students to generate data. As students work, you may need to remind them to *be organized and methodical.* Depending on your students' needs, you may also need to help them record their findings in a table. I choose to intervene only when necessary, without providing too much scaffolding before the need arises.

Students working in groups can be tempted to assign each member a different number to decompose, but this could impede their ability to observe patterns. At this point in the problem-solving process, it would be more effective to have all students write the compositions of all numbers, and simply confer with one another to check.



Check out our website at http://www.mathteacherscircle.org/ for short videos from the author and others about why they love Math Teachers' Circles.

## WHOLE GROUP CHECK-IN #2

### Goal #1: Check for progress.

*"Let's focus on the number 4 since it is not too big, but not too small. Let's begin by collecting the compositions of 4."* Get student responses for the various compositions, and write on the board. *"How do we know that we have all of the possibilities?"* This question's intent is to help students articulate how they are generating compositions systematically. If a student can explain her system sufficiently, she can convince herself and others that she has found all compositions. Instruct students to discuss with their peers how they are sure they've found all compositions of 4. After they have had a few minutes, bring the class back together for students to share their way of knowing. This should lead to a systematic organizational scheme that everyone agrees will produce a complete list.

### Goal #2: Introduce students to an *alternative representation* of the problem.

Draw a 1x4 rectangle next to a few of the compositions of 4, and ask students how we could show each composition with a picture. One possible visual representation is to color the number of boxes corresponding to each number in the composition using alternating colors, or shaded and non-shaded boxes.

### Goal #3: Guide students to choose an organizational scheme.

*"Now, with the data organized (for example, according to the first number or the number of terms), you can be sure that you have found all compositions of each number. In order to more easily manage these lists, make connections, and find patterns, we may consider reorganizing each list differently once we generate it systematically."* Prompt students to make suggestions of organizational schemes, and then decide which scheme the class will use, or each group will use.

*"During the next work phase, focus on finding and recording data according to your chosen organizational scheme. Begin to hunt for patterns. If a pattern seems to*

*be present, decide if it will continue forever. Explore why this pattern accompanies this problem.”*

## WORK TIME/STUDENT SUPPORT

Allot 20 minutes for students to work. Support groups or individuals using the following questions:

1. As students become confident that they've listed all of a number's compositions, ask each student how he can be sure that he has generated all possibilities. This will promote conversation about each student's scheme for generating a complete list of compositions.
2. As students record their findings, ask how they are keeping track. Guide students who need help creating a chart. Help students to focus on the number of total compositions within each part of their chart.
3. Once a student sees a pattern, ask her to think about whether that pattern will always continue. How can she be sure? This question is intended to bring about discussion of what is causing each pattern to persist. Once that is explained, one can argue that it will continue forever.
4. There are two patterns to notice and explain within the table organized by first number. Any student using this organizational scheme should be prompted to find and explain both. (Further discussion is provided in the solutions online at **http://www.mathteacherscircle.org**.)
5. Any student who finds the patterns and describes their underlying cause for one organizational scheme can be prompted to do the same for another organizational scheme.

## DEBRIEF/WRAP-UP

Bring the class together to debrief and share out. Lead a discussion of why each pattern observed holds true for this problem. See explanations on the solution pages at **http://www.mathteacherscircle.org** if desired.

In the debrief, I suggest that you explicitly refer to, name, and record all of the problem-solving strategies that were effective in this investigation. These strategies were listed in bold italics throughout the instructional plan. Highlight for the students that we could only have mastered this problem by organizing our lists. Also, different organizational schemes led to different discoveries. Organization is a very important skill for problem-solving.

## EXTENSIONS

There are many patterns to observe and explain that can be seen through organizing the compositions for each number differently. Some organizational schemes with more advanced patterns are to organize compositions by the highest term, or by the number of 1's used. Other extensions include altering the rules of decomposing. What if only odd terms are allowed? What if there are two types of 1's, such as red 1's and blue 1's?

## TEACHER REFLECTION

I chose to develop a lesson for this problem because I think the systematic generation of data and effectively organizing those data are immensely important problem-solving practices. I was particularly inspired by this problem's wide variety of organizational schemes and different patterns that can be observed from each.

I learned this problem from my friend and mentor, Joshua Zucker. In addition to being deeply involved in Math Circles for both teachers and students, he is the founding director of the Julia Robinson Mathematics Festivals (**http://juliarobinsonmathfestivals.org**), where this problem has also appeared. Joshua played a large part in helping me address this problem thoroughly and write this lesson plan. ▣

# Pizza, Pipe Cleaners, and Lucky Numbers:
# MATH FROM THE CIRCLES

**What has your MTC been up to lately? Here are some mathematical highlights of MTC activities across the country, selected from meeting surveys and online postings. Tell us about your own MTC's highlights by contacting us at [circles@aimath.org](mailto:circles@aimath.org) so that we can add you to the meeting surveys list.**

## Curvature and Strength

**Math Teachers' Circle of Hawai'i ([http://math.crdg.hawaii.edu/match/](http://math.crdg.hawaii.edu/match/))**
**Meeting Date: Saturday, October 4, 2014**
**Session Leaders: Michelle Manes and Mitchell Walker**
This session was inspired by a recent article from Wired ("How a 19th Century Math Genius Taught Us the Best Way to Hold a Pizza Slice," [http://www.wired.com/2014/09/curvature-and-strength-empzeal/](http://www.wired.com/2014/09/curvature-and-strength-empzeal/)) The article asks, "Why does bending a pizza slice help you eat it?" and explores a "surprising geometrical link between curvature and strength." Teachers built bridges from ordinary paper that would be strong enough to hold lots of pennies.

*– Michelle Manes*

Photo: iStockPhoto

# Numbers: Some Surprising, Some Mysterious, Some Lucky

**Tulsa MTC** (http://tmtc.utulsa.edu)
**Meeting Date: Thursday, November 6, 2014**
**Session Leader: Bill Coberly, University of Tulsa/Tulsa MTC**
We explored the surprising properties of some numbers related to the centennial of the birth of the late Martin Gardner, a Tulsa native, whose Scientific American monthly feature "Mathematical Games" and other writings popularized mathematics for over 50 years. Resources included "Lucky Numbers and 2187" by Martin Gardner (The Mathematical Intelligencer, Spr. 1997, 19(2), p. 26), "Friedman numbers" from the website Numbers Aplenty (http://www.numbersaplenty.com/set/Friedman_number/), and "Mysterious Number 6174" by Yutaka Nishiyama (+plus magazine, 2006, http://plus.maths.org/content/mysterious-number-6174).

*– Marilyn Howard*

# How Many Faces Does a 10-Dimensional Cube Have?

**Boston Math Teachers' Circle** (http://bostonmathteacherscircle.wordpress.com)
**Meeting Date: Saturday, September 20, 2014**
**Session Leader: Jameel Al-Aidroos**
We passed out an assortment of pipe cleaners of a variety of colors for teachers to construct models in various dimensions, and viewed a number of websites which showed off a variety of perspectives of hypercubes. Even though a mathematical concept might be considered relatively simple for a research mathematician who has thought about or worked with the concept for a long time, it's important to give a "relaxed" amount of time to a group of teachers who haven't been exposed to the ideas behind the concept in order for them to really enjoy "pulling apart" and discovering the math ideas themselves. The journey is the goal, in some sense, and the more time that teachers can be given to tease apart ideas on their own, the better. I'd expected the essential ideas would just take a half hour or so to expose, but we had a very lively session of almost two hours — teachers enjoyed discovering a number of generalizations as well.

*– Andrew Engelward*

# Volumes Making Full (Math Teachers') Circle

**San Diego Math Teachers' Circle**
(http://www.sdmathteacherscircle.org, https://www.facebook.com/SanDiegoMathTeachersCircle)
**Meeting Date: November 15, 2014**
**Session Leader: Yana Mohanty**
We presented an example of how a discussion between teacher and mathematician found its way into the teacher's classroom. Cynthia's mention of many teachers' concern with teaching volumes inspired Yana to lead a session on volumes last year. Cynthia then successfully used some of the ideas from that session in her classroom. Cynthia's description of the challenges her students faced with volumes got Yana thinking about some more volume problems that may help teachers in the classroom. These problems were presented in this session. If you have used some ideas from the Math Teachers' Circle in your classroom on any topic, we'd love to hear from you, too!

*– Yana Mohanty*

# Judith Covington Receives 2015 Haimo Award

Judith Covington, Professor of Mathematics at Louisiana State University-Shreveport and co-founder of the North Louisiana MTC, has been selected as a 2015 recipient of the Deborah and Franklin Tepper Haimo Award. The Haimo Award is presented by the Mathematical Association of America in order to honor college or university teachers who have been widely recognized as extraordinarily successful and whose teaching effectiveness has been shown to have had influence beyond their own institutions. Recipients of the Haimo Award receive $1,000 and a certificate of recognition, and are invited to present in a special session at the Joint Mathematics Meetings. Covington's presentation, entitled "Problem Solving in the Classroom," will focus on the importance of problem solving to teaching. Her abstract reads, in part, "I will reflect on my experiences with integrating problem solving into math courses for future K-12 teachers. As a result of the increased need to fully engage pre-service teachers in problem solving in my classroom I was led to create the North Louisiana Math Teachers' Circle. I will share my experiences in creating and sustaining this successful Math Teachers' Circle that focuses on problem solving." ⊑

# Fun with Triangular Numbers

Elsa Medina, Richard Grassl, and Mary Fay-Zenk published an article entitled "Fun with Triangular Numbers" in the September 2014 issue of Mathematics Teaching in the Middle School. The article describes the authors' experience running a summer Triangular Numbers Math Camp for 7th and 8th graders and presents several sample problems. Grassl, who is Professor Emeritus at the University of Northern Colorado and a member of the MTC Advisory Board, and Fay-Zenk, a former middle school teacher and administrator in Cupertino, CA, and a co-founder of the MTC model, met through their MTC connection when both retired to the same area of California. They now work together with local children on mathematics and collaborate frequently with Medina, a professor at Cal Poly San Luis Obispo. ⊑
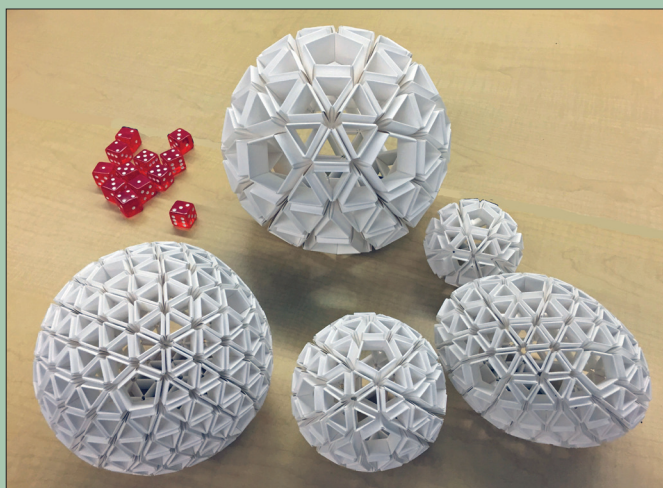
CIRCLE SNAPSHOT

## Paper Polyhedra

*San Diego MTC participant Dave Honda, a teacher at Thurgood Marshall Middle School in San Diego, made the paper sculptures pictured at left as a thank you gift for the Circle's organizers. "They're made entirely of interlocking folded pieces of paper," Organizer David Patrick explained. "The dice in the photo are pictured to provide a sense of scale." For more on paper polyhedra and other mathematical origami projects, visit Mathigon: The Mathematics Education Project at http://www.mathigon.org/origami/.* ⊑

# Alan Schoenfeld Wins 2014 Dolciani Award

Alan Schoenfeld, the Elizabeth and Edward Conner Professor of Education and Affiliated Professor of Mathematics at the University of California at Berkeley, and a member of the MTC Advisory Board, was awarded the 2014 Mary P. Dolciani Award by the Mathematical Association of America. "His insights through more than 30 years of exemplary research have had real and lasting effects on research methodology, teacher education, mathematics curriculum, and the assessment of mathematical understandings," reads the award citation. The Mary P. Dolciani Award recognizes a pure or applied mathematician who is making a distinguished contribution to the mathematical education of K-16 students in the United States or Canada. The award is named for Mary P. Dolciani Halloran (1923-1985), a gifted mathematician, educator, and author, who devoted her life to developing excellence in mathematics education. First given in 2012, the award carries with it a $5,000 prize, made possible through a gift from the Mary P. Dolciani Halloran Foundation. Schoenfeld is a fellow of AAAS and AERA and the 2011 winner of the International Commission on Mathematical Instruction's Felix Klein Medal. ▣

# Diana White Named Director of the National Association of Math Circles

Diana White, Associate Professor of Mathematics at the University of Colorado Denver, was recently named Director of the National Association of Math Circles, based at the Mathematical Sciences Research Institute in Berkeley, CA. White founded the Rocky Mountain MTC in 2009 and was one of the first researchers to examine how MTCs affect participating teachers. She has been a facilitator at a "How to Run a Math Teachers' Circle" workshop and is a Co-Principal Investigator on a National Science Foundation grant, "Investigating the Impact of Math Teachers' Circles on Mathematical Knowledge for Teaching and Classroom Practice." White speaks frequently about MTCs at national and international conferences and is particularly interested in helping mathematicians begin working with teachers. ▣

# MTC Article Featured in the Notices of the AMS

An article entitled "Math Teachers' Circles: Partnerships between Mathematicians and Teachers," by Brianna Donaldson, Michael Nakamaye, Kristin Umland, and Diana White, appeared in the December 2014 issue of the Notices of the American Mathematical Society (**http://www.ams.org/notices/201411/rnoti-p1335.pdf**). The article describes key features of the MTC model and includes a summary of key research findings and benefits for teachers and mathematics professors who participate in MTCs. According to the article, Math Teachers' Circles "leverage mathematicians' disciplinary knowledge and skill" and "encourage teachers to develop as mathematicians," with the goal of creating an "ongoing professional K-20 mathematics community." Work on the article was supported by National Science Foundation grant Discovery K-12 1119342. ▣

# Dispatches from the Circles
## Local Updates from Across the Country

## Connecticut • 🚩

People's Bank renewed their grant of $1,000 to Rosemary Danaher for our **Fairfield County Math Teachers' Circle** meetings this year.

*– Hema Gopalakrishnan*

## Michigan • 🖐

The **Wayne County Math Teachers' Circle** received a National Association of Math Circles (NAMC) mini-grant from MSRI.

*– Nina White*

## New York • 🟩

**New York Math & Dinner** Leader Japheth Wood gave the keynote talk at the Queens College TIME2000 conference on Friday, November 14, 2014 (**http://time2000.qc.cuny.edu/conference/50/conference**). Last year's keynote address was given by James Tanton (**http://time2000.qc.cuny.edu/conference/49/past-conference/pastconference**).

*– Japheth Wood*

## North Carolina • 🔻

**Smoky Mountain MTC** has been awarded a seed grant and an exchange grant from the NAMC.

*– Sloane Despeaux*

## Oklahoma • 🟦

**Tulsa Math Teachers' Circle** member Donna Farrior has successfully initiated a Circle dedicated entirely to middle school girls, with about 20 regular attendees: Tulsa Girls' Math Circle (**http://tgmc.utulsa.edu/**). Special thanks to Bill Coberly, Ellen Eisenhour, and Carter Scarborough for their help on this project. Also, two members have recently received awards: Lynette Shouse of Grissom Elementary School won the Tulsa Public Schools Teacher of the Year Award for 2013-2014, and Sarah Hagan of Drumright High School has been selected as one of NPR's 50 Great Teachers.

*– Marilyn Howard*

## Welcome to Our New Circles

- Wayne County, MTC (Detroit, MI): **http://www.math.lsa.umich.edu/WCMTC/**
- Mississippi Delta MTC (Cleveland, MS)
- Corpus Christi, TX
- Smoky Mountain MTC (Cullowhee, NC): **https://www.facebook.com/sm2tc**
- Edwardsville, IL
- Black Swamp MTC (Bowling Green, OH)
- Chicago, IL
- Morrow, GA
- Dubuque, IA
- Emory, VA
- Ft. Thomas, KY
- Glass City MTC (Toledo, OH) 🟩

# Problem Circle: Enigmatic Pi
## by Paul Zeitz

With Alan Turing making appearances in print and on the screen this winter, we find ourselves in the mood for some cryptography. And we are always in the mood for $\pi$! So, here is a riddle: Why does the fraction
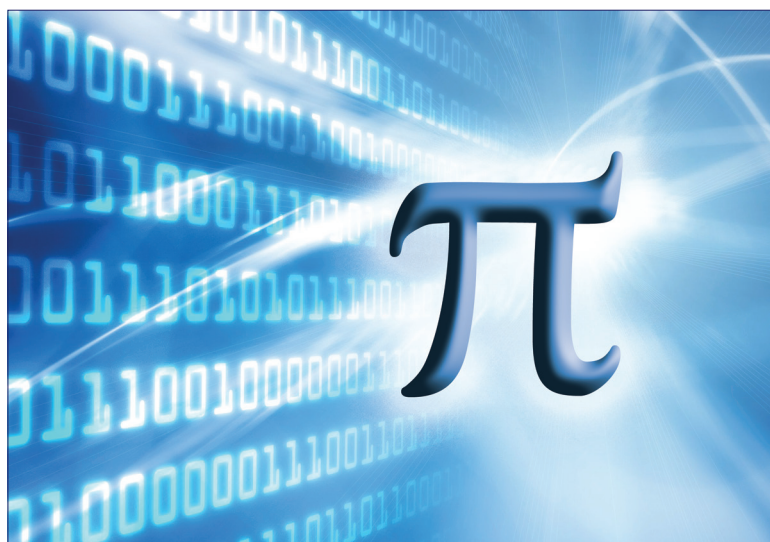
1465015015545333800853587426164919420547705739574735735699614506779627235211658993226748473396937157656378266582913149004617644403561994854413432341027654775004762468657417226352100158301330227771835175451338194599850197745825719598218994781188135847935/$2^{841}$

make us think of $\pi$?

Hint #1: This fraction is approximately equal to 0.99951553344089294479799277790673806587…, so it certainly is not equal to $\pi$! However, if you write it in base-2 (binary), it will TERMINATE and be exactly 841 digits long.

Hint #2: We were inspired by Carl Sagan's ideas about communicating with aliens, which he wrote about in non-fiction books and essays, as well as his science-fiction novel Contact, which was made into a movie starring Jody Foster in 1997.
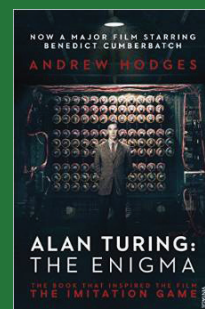
*This puzzle is used by permission from Paul Zeitz and first appeared on the website of the Proof School, a new independent secondary school in San Francisco for kids who are passionate about mathematics. You can view the original post at* [http://proofschool.org/2014/03/22/a-sci-fi-number-riddle-for-pi-month/](http://proofschool.org/2014/03/22/a-sci-fi-number-riddle-for-pi-month/).

Binary: Flavio Takemoto, freemages; Tickets: iStockPhoto; Book Cover: Courtesy of Princeton University Press

# Liar's Bingo
## by Bob Klein and Steve Phelps

Patterns are one of Math Circles' great levelers. From recognizing a pattern to generating terms, to abstracting and making inferences, tasks based on patterns embody the "low-threshold, high-ceiling" trait of good problems. Liar's Bingo is all about patterns, and we have used it with kids aged 11 to 75 years old. We've had a Grade 3 special education teacher work side-by-side with an AP Calculus teacher to understand what makes it work, with each teacher finding challenges and rewards in the problem.

Liar's Bingo is played with strips of paper containing a 1 x 6 array of positive integers, some colored black and some red. Six sample strips are included in the figure at right (one strip per row). A complete set may be found at **http://mathteacherscircle.org**; these should be cut into individual strips and shuffled before using. Begin by passing out handfuls of strips to groups of participants. Ask them to "Put the strips in order." This request often leaves participants dumbfounded for a moment: "What order should we put them in?" or "Do you mean numerical order?" are typical questions. We are purposefully vague at this point, for we want the participants to study the cards and look for underlying patterns themselves.

Have the participants start working in pairs so they can "huddle" around the strips, and follow this with table discussions of 4-6 participants. Then, with the whole group, record found patterns on a chalkboard or chart paper for all to see. Take a moment to note what patterns you find in the figure at above right.

We have led this activity for groups with wide-ranging prior experience and have been surprised each time to have participants discover patterns that are new to us. Typically, groups will offer ideas related to the pattern of reds versus blacks, parity and order (increasing versus decreasing, for instance), place-value, and maximum and minimum values. The better

| 41 | 21 | 11 | 5 | 3 | 0 |
|----|----|----|----|----|----|
| 17 | 77 | 47 | 53 | 55 | 56 |
| 31 | 51 | 61 | 75 | 73 | 70 |
| 16 | 76 | 46 | 52 | 54 | 57 |
| 55 | 35 | 5 | 11 | 17 | 14 |
| 34 | 54 | 64 | 70 | 76 | 75 |

sessions we've run have resulted from following Joshua Zucker's suggestion to "be less helpful"—participants get a long way when encouraged to struggle.

After charting a list of patterns, the facilitator announces that it is time to play Liar's Bingo. The rules of the game are simple: the facilitator will ask a participant to read (left-to-right) the sequence of six colors but to lie about exactly one of those colors. For instance, in the case of the first strip shown in the figure above, a participant might decide to lie about the fourth entry and say "black-black-black-red-black-red." The facilitator, without looking at the participant's strip, will divine that the participant has lied about the number 5 on the strip. The announcer will confirm that the facilitator was correct, to the amazement of the room. People generally ask to test the facilitator on another five or six strips.

The job of the room then becomes to figure out how the "trick" works. The facilitator becomes a black-box tester for sequences suggested by the participants. Some questions that help groups get unstuck include: "Independent of the strips in front of you now, what would be an interesting sequence of colors to ask about?" or "What sequence of colors would get me

to say 54? How about 45?" Eventually, people want to know about all-red or all-black strips, strips with all black except the rightmost entry, etc. Prospective facilitators are advised to let the groups struggle despite the strong desire to push them toward the answer. In our experience, the room gets there and the activity has a natural flow for about 1.25 hours.

Once people at a table "get the trick," their role shifts to first being "tested" by the table to see if they are able to replicate the trick and then to helping others discover the trick. One of the authors, at this point, finds it useful to wander the room announcing, "No spoilers! Your job is to help others discover for themselves."

To facilitate that same joy of discovery here, we offer the following "key" values based on playing the game with the strips in the figure (with one number's color lied about on each strip):

| | |
|---|---|
| B-B-B-B-B-B | 0 |
| R-R-R-R-R-R | 77 |
| B-R-R-B-B-R | 31 |
| R-B-B-R-R-B | 46 |
| B-B-R-B-B-R | 11 |
| R-R-R-B-B-B | 70 |

***Spoiler alert! Before reading on, you may wish to pause to contemplate the explanation for the trick.***

The underlying mathematics isn't that difficult and involves a mixture of decimal and binary representations. The first six entries on the strip represent the tens-digit of the number being lied about, but represented in binary. Hence the string beginning "B-R-R" would be 011, or 2 + 1 = 3. The last three entries on the strip represent the one's-digit being lied about, so "B-B-R" would be 001 or 1. This explains why the strip B-R-R-B-B-R above is coded "31."

Some great questions to explore at this point: Can you reverse-engineer a strip? Could you create your own set? How many strips could be constructed in which "31" is the number being lied about? How many strips make a complete set? What would happen if someone lied about two colors? Are variants possible using eight entries per strip? Other bases?

"Closing" the activity is important because many of the participants will now be so focused on the "trick" that they will lose sight of the pattern recognition work at the beginning. Have the room revisit the patterns charted at the front of the room in the context of their new understanding of Liar's Bingo. This will easily push the session to two hours or more and may have to be assigned as "take-home" play.

Liar's Bingo is accessible to a wide audience and has, in our experience, hit that narrow band of "perplexity" or, to use another Circles' excellent phrase, "funstration," that gets under participants' skin yet needles them to persist to understand the problem. The "magic trick" performance is an important and memorable feature of the activity, and its occurrence near the middle of the session recharges the group's motivation. While teachers find this to be great fun, students love it, too, and sets of Liar's Bingo cards have been seen circulating school hallways in Puerto Rico, Thailand, Ohio, and the Navajo Nation.

*Bob Klein teaches at Ohio University and is co-founder of the South East Ohio MTC. Steve Phelps teaches at Madeira High School and is co-founder of the Cincinnati Math Teachers' Circle. Phelps was first introduced to Liars' Bingo in a presentation by Chuck Sonenshine in a 1990 methods course. When asked for a copy of his cards, he suggested that Steve make his own, which he did that evening. You can find out more about "Mathemagician" Chuck Sonenshine on his website at http://www.keynotemath.com/.*

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