

Math Teachers' Circles - Impacting
Teachers' Mathematical Knowledge
for Teaching.

Diana White
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Joint Math Meetings
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What is a Math Teachers' Circle?

- Professional Development Program for Middle-Level Math Teachers
- Focused on Mathematical Problem Solving
- Over 23 active MTCs in U.S., dating back to 2006, with 10-12 more expected to launch in Summer 2011
- Basic Format
 - Summer Immersion Workshop
 - Academic Year Follow-up Sessions

Next: Typical Problem

Typical Problem

- Write numbers from 1 to 100 on the board. Select any two of the numbers, erase them, and write on the board the sum plus the product of the two numbers. For example, if you erased 2 and 5, the sum plus the product is 7 plus 10, or 17, and so you write a 17 on the board. Now there are two 17s, but that's OK. Repeat this process of selecting two numbers and replacing them with their sum plus their product. What are the possible outcomes?

Next: Aspects

Key Aspects of Problem

- Problem Solving Techniques/Skills
 - Ask a simpler question
 - Use fewer numbers
 - Use simpler operation
 - Generalize
- Connections to K-12 mathematics
 - Commutative and associative properties
 - Structure and symmetry
 - Algebraic Representation

Next: MKT

Mathematical Knowledge for Teaching

“By *mathematical knowledge for teaching*, we mean the mathematical knowledge used to carry out the work of teaching mathematics.” (Hill, Rowan, & Ball, 2005).

■ Examples

- Explaining terms and concepts to students
- Analyzing students' solutions
- Judging and correcting textbook treatments
- Using representations accurately in the classroom
- Providing students with examples of mathematical concepts, algorithms, or proofs

Next: Analyzing Student Solutions

Analyzing Student Solutions

- **Determine if each student is using a method that could be used to multiply any two whole numbers.**

Student A	Student B	Student C
$\begin{array}{r} 35 \\ \times 25 \\ \hline 125 \\ +75 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 175 \\ +700 \\ \hline 875 \end{array}$	$\begin{array}{r} 35 \\ \times 25 \\ \hline 25 \\ 150 \\ 100 \\ +600 \\ \hline 875 \end{array}$

Next: Analyzing Student Solutions

Analyzing Students Solutions

- There were 64 teams at the beginning of the NCAA basketball tournament. There are 5 starting players on each team. How many starting players were there at the beginning of the tournament?
 - That would be 64 times 5. I use one 10 because I know 5 times 10 is 50. Then you do that six times. That's 30, I mean 300. Then you add 4 five times, which is 25, no 20. I added it all together and got 320.
 - 64 means $60+4$. So I did 60 five times, for 300. Then 4 times 5 is 20, so the answer is 320
 - I split 64 into four parts – 20, 20, and 20. I did each separately: 20 times 5 is 100, 20 times 5 is 100, 20 times 5 is 100. Then the last part, 4 times 5, is 20. All together 320.

Next: Research Questions

Research Questions

- Does participating in a Math Teachers' Circle Immersion Workshop result in increases in teachers' Mathematical Knowledge for Teaching?
- Does increased practice with and skills for mathematical problem solving increase teachers' Mathematical Knowledge for Teaching?

Next: Study Description

Study Description

- Administered Learning Math for Teaching Instrument
 - Subscales used:
 - Number Concepts and Operations
 - Geometry
 - Pre-post format during 1 week workshop (M-F)
 - 49 teachers at 3 sites – all (sub)urban, with a few rural teachers

Next: LMT Instrument

Learning Math for Teaching Instrument

- Developed at University of Michigan
- Extensively tested to establish psychometric soundness
- Has been linked to increased student achievement in a large scale study (Hill, Rowan, & Ball, 2005)
- Scores are standardized to a normal distribution with mean 0 and standard deviation 1

Next: Study Results

Study Results

- Number Concepts and Operations
 - Mean increased by .34 standard units
 - P-value less than .0001
- Geometry
 - Mean increased by .14 standard units
 - P-value of .059

Next: Study Implications

Study Implications

- Preliminary evidence that MKT is positively impacted by MTCs
- Preliminary evidence that MKT can be increased via development of mathematical problem-solving in teachers
- Need not always teach MKT directly

Next: Future Directions

Future Directions and Goals

- Replicate study in summer 2011, expanding to more MTC sites and replacing the geometry subscale with the proportional reasoning subscale
 - Case studies of teachers who participate in MTCs – impact on classroom practices, especially with regards to mathematical problem solving
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Mathematical Knowledge for Teaching (MKT)

- “Knowing mathematics for teaching demands a kind of depth and detail that goes well beyond what is needed to carry out the algorithm reliably.” Debra Ball, Heather Hill, and Hyman Bass
 - developing definitions with attention to language
 - analyzing equivalence of two definitions
 - providing key examples and special cases
 - analyzing students' mathematical arguments
 - analyzing students' interpretations of others' arguments
 - distinguishing misconceptions from alternative definitions
 - knowing the distinct conceptual models that correspond to a mathematical operation or structure
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Mathematical Knowledge for Teaching

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Examples:

explaining terms and concepts to students,

- interpreting students' statements and solutions,
 - judging and correcting textbook treatments
 - - using representations accurately in the classroom, and providing students with examples of mathematical concepts, algorithms, or proofs”
 - (Hill, Rowan, & Ball, 2005).
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Common vs. Specialized Mathematical Knowledge

- Individuals can be strong in common mathematical knowledge, but not specialized mathematical knowledge for teaching, and vice versa.
 - Suggests there *is* professional knowledge for teaching mathematics.
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Mathematical Knowledge for Teaching

- “Teachers need to know mathematics in ways that enable them to help students learn. The specialized knowledge of mathematics that they need is different from the mathematical content contained in most college mathematics courses, which are principally designed for those whose professional uses of mathematics will be in mathematics, science, and other technical fields.” (National Research Council 2001)
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CBMS: The Mathematical Education of Teachers

- “The mathematical knowledge needed for teaching is quite different from that required by college students pursuing other mathematics-related professions.
 - Prospective teachers need a solid understanding of mathematics so that they can teach it as a coherent, reasoned activity and communicate its elegance and power. Mathematicians are particularly qualified to teach mathematics in the connected, sense-making way that teachers need.
 - For maximum effectiveness, the design of this instruction requires collaboration between mathematicians and mathematics educators and close connections with classroom practice.”
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CBMS Quotes

- After reading and writing, mathematics is widely viewed as the most important component of K-12 education to promote future success in college and subsequent careers
 - It goes without saying that mathematicians want prospective teachers to have a solid understanding of the mathematics that they will teach. The daunting challenge is how to achieve this goal, given the diverse mathematical preparation of college students who will become teachers and the changing views about what mathematics knowledge is needed to be an effective teacher.
 - It is by now widely acknowledged that many practicing teachers were not adequately prepared by the mathematics instruction they received to meet these challenges. As K-12 students – often in the primary grades – they lost their curiosity about mathematics. When the rules and procedures one is taught are not conceptually anchored, memorization must pass for understanding, and mathematics becomes an endless, senseless parade of disparate facts, definitions, and procedures.
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Judging and Correcting Textbook Treatments

- An *equilateral* triangle is a triangle with sides of equal length, angles of equal measure, and where each interior angle is 60 degrees
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Analyzing Students Solutions

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Using Representations in the Classroom

