



Making Change Not Every Pattern Leads to a Solution 65 Uses for a Paperclip Conjectures & Counterexamples Turn Your World On Its Side Symmetry in Scanning Codes

### A NOTE FROM AIM

### **Rich Histories**, **Bright Futures**

Dear Math Teachers' Circle Network,

This issue's feature articles focus on problems with rich histories that make for engrossing MTC topics. "The Roommate Game," by Emily Dennett and Chris Bolognese, draws inspiration from a 50-year-old article to develop a nicely structured investigation of a type of stable matching problem, the stable roommate problem. (Be sure to check out their excellent accompanying resources if you are considering doing this session with your own MTC!) "Recruiting Change for a Dollar" by Craig Collins, Elizabeth Donovan, and Cynthia Kramer, describes how the authors used another classic problem—counting the possible ways of making change for a given amount-during recruitment sessions for their new MTC. It is an approachable problem with a quick set-up that nevertheless leads to some interesting and deep questions even within the span of a "demo" session.

Dan Finkel's "65 Uses for a Paperclip" has appealing historical resonance even within our MTC community. His use of a simple grid of lattice points to generate questions for investigation brings to mind Tatiana Shubin's classic "Grid Power" session, in which she begins by giving participants 7 minutes to write down questions inspired by staring at a sheet of grid paper. (You can see Tatiana facilitating this session in our Video Library at **mathteacherscircle.org**.) Tatiana's session, which she began developing more than 10 years ago, is the first MTC session I can remember that put a strong emphasis on problem posing. Dan's article builds on that foundation by introducing two powerful but easy-to-implement techniques: a gamified version of a key mathematical process that he calls "Conjectures and Counterexamples," and three surprisingly simple steps to support a "Thinking Classroom," drawn from the research of Peter Liljedahl.

MTCs across the country continue to thrive. It's especially exciting to see new MTCs grow near established ones, and also to witness the accomplishments of the teams who attended the MTC training workshop at the MAA last summer, including a major grant landed by the team from Oklahoma. I'm also proud to announce that an article describing case studies of MTC-participating teachers was recently published in the open-access journal The Mathematics Educator ("News and Views"). My co-authors and I are hopeful that the article will prove useful and will provoke some good discussion within the MTC community.

Happy problem solving, and here's to more Thinking Classrooms in 2019!

Buanna Donaldson

Brianna Donaldson, Director of Special Projects

American Institute of Mathematics

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## 65 USES FOR A PAPERCLIP

### Conjectures and Counterexamples in a Thinking Classroom by Dan Finkel

have been developing a structure to help teachers invite students into a genuine mathematical process, starting with their own understanding. We call it **Making and Breaking Conjectures**. Recently, we talked about this structure in our Math for Love/WXML Math Teachers' Circle in Seattle. First, what are conjectures? What are counterexamples? A **conjecture** is a mathematical hypothesis, a guess of the underlying structure or pattern based on what we know so far. A **counterexample** is an example that proves a conjecture false. Mathematics as a field progresses by way of conjectures and counterexamples. The good news is, we can use them even with very young kids.

We played the game **Counterexamples** to get a sense of how this works. The game is super-simple: The teacher makes a false conjecture, and the students prove it false with a counterexample. For instance:

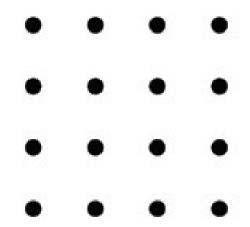
Teacher: All pets have four legs.

- *Students:* No! Because birds have two legs! *Teacher:* Okay – refined conjecture: All pets have two or four legs.
- *Students:* What about a snake?
- *Teacher:* That's a pet with no legs. So I'll refine my conjecture again. All pets have at most four legs. *Students:* What about a spider?

And so on. So compelling is the game that our MTC almost got off track when we considered examples of conjectures about area and perimeter. But there was work to be done on generating our own conjectures.

To warm up our observing, noticing, wondering, and conjecturing muscles, we started by asking, "What can you do with a paperclip?" This is a classic exercise, and kindergarteners tend to beat the pants off of adults. After spending ten minutes or so, groups had come up with everywhere from 20 to 65 uses for a paperclip. That was just the warm-up, though, so we didn't go too deeply into what the uses actually were; the main event was still on its way.

#### What Can You Do With This Grid?



Considering this question was trickier, and required a little more discussion to draw out observations and questions. By the time the conversation was done, however, we had a tidy collection of questions to consider:

- How many rectangles can be formed by connecting dots with straight lines?
- How many sides could a polygon have on the grid? Could it include all 16 dots?
- Can you make a square with any number of dots on its perimeter?
- How many different paths are there from the bottom left to top right?
- How many lines would there be if you connected every dot to every other dot?
- How many different lengths can you find by connecting two dots?
- How many symmetric shapes can you make with corners on the dots?
- How many fractions can you represent on the grid?
- How many different areas can you get if you form a triangle with three points on the grid?
- Can you find two triangles with the same area that are not similar?





Take in that list for a minute. There are weeks of beautiful, high-level problems to explore here. For each, you can start by casting around. Soon, you'll find you have conjectures. Once you have a conjecture, you can try to break it right away by looking for a counterexample. Refine and repeat until you end up with something that seems to be true, and then you can put together an argument, with luck and a little insight, into why it actually might be true. We're really doing mathematics!

With all these questions to consider, we wanted to provide some guidelines. So we posed a choice: Teachers could choose the problem that inspired them most and work on that one. Or they could work on the question of how many different areas a triangle could have if its corners are on the grid. (Having a default question is something we've found can help prevent groups from becoming aimless. With an actual classroom, you might want to skip some of these steps, and start from a more tightly focused, teacher-chosen task in the first place. Still, opening up the entire thinking process can definitely be worth it, if you're ready to take the step.)

And with that, we sent the groups out to work on their own. However, we had one more idea ready at hand to ensure things went as well as they could go.

#### Transitioning to a "Thinking Classroom"

Peter Liljedahl (<u>http:/www.peterliljedahl.com/</u>) has been developing a series of concrete steps to change the classroom to support genuine thinking in mathematics. The steps to encourage this transition are bizarrely simple to employ. Here's how you begin, according to Liljedahl:

- Use "tasks" (that is, make real, meaningful mathematical experiences the heart of what you ask students to do).
- Use vertical, erasable surfaces (i.e., whiteboards).

• Assign groups in a visibly random fashion. It seems almost too easy, but the room was electric with thinking energy. It's hard to overstate how getting everyone standing and working together impacts the quality of thinking and engagement in the room. One of Liljedahl's recommendations is to use just one marker per group. You can see the value of that suggestion in one of the videos in the blog post version of this article, when the carrier of the marker is drawn back up to the board to write in details of someone else's computation.

With so many teachers considering such a range of problems, we decided to let groups pair up to share what they had discovered with each other.

All in all, everyone was totally energized by the workshop. The participants left with a whole bunch of great questions to ponder about grids, and the motivation to use tasks, whiteboards, and visibly random groups in their own classrooms.

And we organizers came away with a goal for this year's meetings, especially for the middle school level: Create a series of professional development sessions that will help everyone turn their classroom into a Thinking Classroom, where students use Conjectures and Counterexamples to power genuine mathematical experiences.

Dan Finkel is Founder and Director of Operations of Math for Love, and co-organizer of the WXML/Math for Love Math Teachers' Circle in Seattle. This article is adapted from a blog post at <u>http://mathforlove.com/</u> <u>2017/08/math-teacher-circles/</u>.

## Recruiting Change for a Dollar

ecruitment is a key component to beginning a Math Teachers' Circle. Our location in rural western Kentucky guarantees most teachers will come from a radius of 50 miles, making substantial travel time for those participating in our circle. Before we launched with an immersion workshop in July, we felt it would be beneficial to recruit by visiting local schools. So, we offered fall and spring recruitment workshops at a regional middle school aimed at giving participants a taste of our future meetings.

### Making Change

We began the spring workshop by waving a dollar bill and asking the question, "Can anyone give me change for a dollar? I'd like to get a snack from the vending machine." After an initial scramble of people looking for their wallets, the participants were presented with bags of change that included half dollars, quarters,



Workshop participants explore the problem with provided bags of coins.

dimes, nickels and pennies. Different coin combinations were suggested and recorded on the board. This allowed us to pose the overarching question of the meeting: How many different ways are there to make change for a dollar?

Due to the sheer size of the task we dialed the problem back and began by asking the teachers to make change first for a nickel and then a dime. A discussion ensued on what it meant to make change: Must the amount be broken into smaller denominations, or can one simply (ex)change one coin for another? Is a dollar coin change for a dollar bill? In the end, the group decided as a whole that making change required smaller valued coins to be traded for a larger denomination.

We continued the activity and found change for a quarter as well as a half dollar. We discussed the strategies implemented by the various groups. One common approach was to use as many large-valued coins as possible and trade them out for smaller denominations. A second method was to use as many coins as possible first. Thus, some of the teachers approached the solution to the problem at hand with a top-down approach while others worked in the opposite direction. Initially everyone began by writing out the words for each coin, but they soon began using abbreviations, like *P* for pennies or *N* for nickels, and tracking data in a table in order to keep track of the ever-growing lists.

### Patterns Don't Solve Everything

There's one catch to this family of problems: There is no discernible pattern to the growth in the number of ways to make change as the desired amount increases from a nickel to a dollar. During the process, several patterns were discovered when making change systematically, for example the orderly "blocks" shown in Figure 1. While these patterns were helpful in creating the lists, extrapolating the results did not allow the participants to solve the original problem. Thus, a seemingly simple question quickly led to an atypical scenario: The use of low-level scaffolding questions resulted in neat and satisfying patterns, but these were ultimately insufficient to solve the larger problem as the target amount

Dimes	Nickels	Pennies
2	1	0
2	0	5
1	3	0
1	2	5
1	1	10
1	0	15
0	5	0
0	4	5
0	3	10
0	2	15
0	1	20
0	0	25

was scaled up. (See Figure 2.)

Once the final solution was reached, we discussed the answer for making change for \$2, \$5, and so on. We briefly introduced the notion of a generating function to consider. The generating function gives a unique way to determine the number of ways to make change for any amount without writing down every

*Fig. 1: Twelve ways to make change for a quarter.* 

possibility, thereby giving a pattern to the seemingly unexplained growth. (Frank Morgan's MAA Math Chat blog post "293 Ways to Make Change for a Dollar" is a great starting point for those who wish to delve further into the mathematics!)

#### Change-ing the Problem

From here new problems were considered. For example, in how many ways can you make change for a dollar using exactly 50 coins? What is the maximum amount of money one can have without having change for a dollar? The immediate response to this last question was "99 cents!" However, participants soon realized that although one can only have 99 pennies without having change for a dollar, it is possible to have a collection of coins worth more than a dollar and yet not have change for a dollar. To our surprise the participants quickly determined the solution to this question as a group. One member suggested starting with three quarters, and then additional lesser-valued coins were added to the running total. The final value of \$1.19 is comprised of three quarters (or one halfdollar and one quarter), four dimes, and four pennies. Given the difficulty with finding change for a dollar using exactly 50 coins, the participants were given a "homework" question: In how many ways can change for a dollar be made with exactly 19 coins? A Google Classroom was created to continue the discussion at a distance and to continue exchanging ideas on finding a

solution. We believe it will prove to be an invaluable resource for maintaining connections among teachers in our highly rural area.

#### Reflection

Overall we felt these types of monetary problems provided engaging, classroom-adaptable entry points for teachers who were

Unit of	Number of Ways	
Currency	to Make Change	
1¢	0	
5¢	1	
10¢	3	
25¢	12	
50¢	49	
\$1	292	
\$2	2,728	
\$5	111,022	
\$10	3,237,134	
\$20	155,848,897	
\$50	58,853,234,018	
\$100	9,823,546,661,905	
	Fig. 2	

Fig. 2

new to MTCs. Something that we particularly enjoyed about the activity is that

patterns are often something that we as mathematicians search for in a problem. However, in this case, there is no simple, predictable pattern to build to an answer, encouraging participants to reach outside their comfort zones and ponder alternative strategies in order to make progress.

Craig Collins and Elizabeth Donovan are Assistant Professors, and Cynthia Kramer is an Instructor, in the Department of Mathematics and Statistics at Murray State University. The three are members of the Western Kentucky MTC's founding leadership team.

### References

Brualdi, R. A. (1977) Introductory Combinatorics. New York, NY: North-Holland.

Cluster, N. (2016, July 19) Change for a Dollar? <u>https://bit.ly/2COoAyn</u>

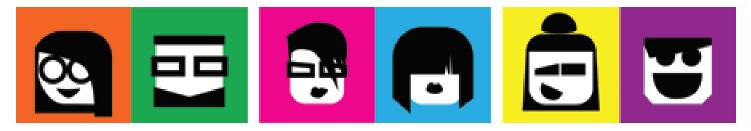
Morgan, F. (2001, April 19) Frank Morgan's Math Chat – 293 Ways to Make Change for a Dollar. <u>https://bit.ly/2CQq6jB</u>

For links to this article's resources and more, visit us at <u>www.mathteacherscircle.org/newsletter</u>.





During her sophomore year at Wittenberg University, this article's co-author, Emily Dennett, was a Resident Advisor (RA) in a freshman dormitory. In August, 28 freshman women moved into the 14 rooms. In May, the same 28 women moved out of the same rooms. No one had left college, and no one had switched roommates. Anecdotally, she knew this was rare, but she wondered if there was a way to mathematically describe this type of stability.



It turns out that David Gale and Lloyd Shapley considered this problem more than 50 years ago. In their 1962 article (reprinted in 2013), "College Admissions and the Stability of Marriage," they proposed the concept of "stable matching":

> An even number of boys wish to divide up into pairs of roommates. A [matching] is called stable if under it there are no two boys who are not roommates who prefer each other to their actual roommates. (p. 388)

In a session of the Columbus Area MTC, we started out by establishing a definition of a "matching," which is a complete set of pairs. We then explored the following concrete situation about roommate matching, in which each person is paired with exactly one other



person. Suppose four women are moving into a dorm: Allison, Bella, Chloe, and Desiree. Each of them ranks the other three women in order of their preference for rooming with that person, with the person with whom they would most want to live listed first. The figure below at left shows each woman's preference list.

Given the preference lists for each individual, can we find a matching that is stable? How many matchings are stable?

It's helpful to rephrase the definition of a matching being stable as follows: Would any pair of these women go to their RA and ask to change rooms because they would rather room together than with their current roommates?

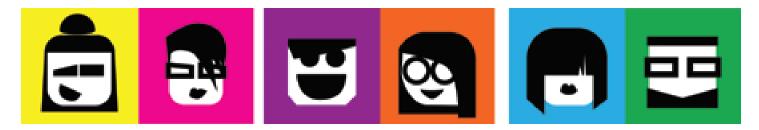
For example, consider the following matching: Allison and Desiree are paired, and Chloe and Bella are paired. Is this a stable matching? You may notice that Allison is paired with her last choice. You also may notice that Chloe prefers Allison to her current roommate. Since Allison and Chloe both put each other first on their preference lists, any matching that doesn't put them together is destined for a meeting with the RA.

However, if Allison and Chloe are roommates, the result is a stable matching. Three of the four have their first choices for roommates, and so there is no pair of women who would both prefer to room with each other rather than their assigned roommates.

### The Exploration

After spending some time making sure everyone understood the definition of a stable matching, we set out to explore new examples to put our matchmaking skills to the test! Each participant received two sets of cards: one set representing eight women who needed to be matched into four pairs, and one set representing six men needing to be matched into three pairs. Each card had the name and picture of a person, along with their preference list.

The preference lists given on the cards were specifically chosen to illustrate different aspects of the stable



roommate problem. The set of cards depicting men did not have any stable matchings, while the set of cards depicting women had three possible stable matchings.

Participants were given 30 minutes to find stable matchings for these two sets. A dry erase board was set up in the front of the room for people to record possible stable matchings. After a matching was added to the board, other groups would work to either validate the matching or find an "unstable" pair. This facilitation technique was especially effective for groups to communicate and critique one another's reasoning.

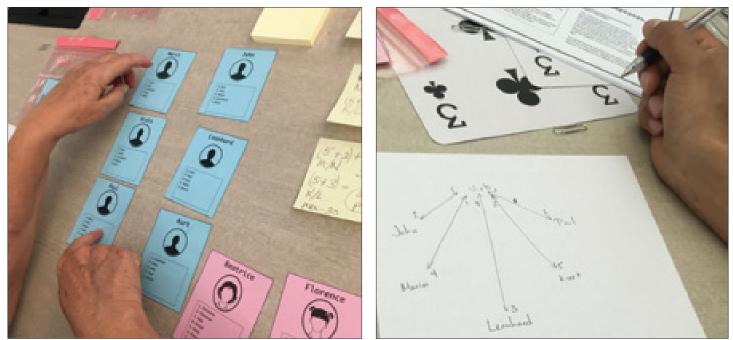
This initial time led to several interesting discoveries, which we discussed together. Some groups had created their own notation by using the physical positioning of the cards to keep track of which preference lists no longer needed to be checked. Others began exploring the problem using graph theory or matrices.

Several questions were generated and recorded after the initial exploration for all groups to consider, including:

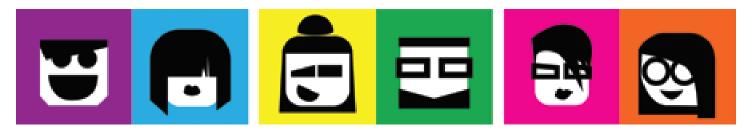
- If one person is paired with their least preferred person, must the matching be unstable?
- Is there always a unique stable matching?
- If there is not a stable matching, can we prove it?
- Is there an algorithm that can be used to find a stable matching?
- Are there orders of stability? If there isn't a stable matching, is one more or less stable?
- What if we have triple rooms (multiples of 3)?

We gave participants another 30 minutes to engage with the questions that most interested them. A computer simulation of the problem coded in Scratch was also introduced. The computer simulation randomly generates new preference lists for each individual so that any algorithms or hypotheses can be tested on a new instance of the problem.

Allowing participants autonomy over what they wanted to investigate was powerful. Some participants developed a scoring metric to investigate questions of stability and orders of stability. Others studied counting problems related to matchings (e.g., how many



Left, a workshop participant explores possible roommate matches using the provided cards. Right, a participant explores possible answers using graph theory.



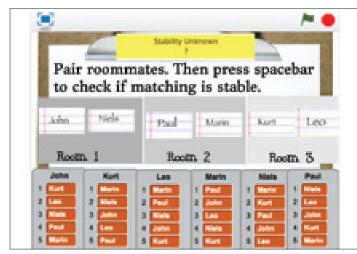
possible matchings are there for 6 people?). Another group worked on developing an algorithm to find a stable matching. Each group was excited to share their findings with the rest of the MTC.

### Reflections

At the end of the session, we connected the roommate problem to related mathematics research and to our teaching practices. We shared the following salient quote about making higher-level mathematics accessible from Gale and Shipley's original article about stable marriage:

> Most mathematicians at one time or another have probably found themselves in the position of trying to refute the notion that they are people with "a head for figures," or that they "know a lot of formulas." At such times it may be convenient to have an illustration at hand to show that mathematics need not be concerned with figures, either numerical or geometrical. (Gale & Shapley, 2013, p. 391)

This session took place during the first day of the Columbus Math Teachers' Circle Summer Immersion Workshop. At the end of the day, participants reflected on the day via videos and surveys, sharing how this activity was useful to them—both personally and professionally.



A sample problem from the online simulator version of the game.

One participant commented in her video that she really appreciated the storytelling element that launched this session. She felt that having a personal connection to the mathematics would be helpful in getting students to persevere in working on a difficult problem and that this activity provided an exemplar of what she will try in her own classroom.

On the surveys, several participants indicated that they were interested in using this activity in their mathematics classrooms. They valued the fact that the problem was relatable and accessible, and could also lead to deep mathematical thinking.

The stable roommate problem proved to be an enjoyable and fruitful activity. We encourage other MTCs to try the problem and share their own experiences!

*Emily Dennett is an assistant professor of mathematics at Central Ohio Technical College. Chris Bolognese, co-founder of the Columbus MTC, is a high school teacher and department chair of Columbus Academy Upper School.* 

### Resources

Gale, D., & Shapley, L. S. (2013). College admissions and the stability of marriage. American Mathematical Monthly. Vol. 120, pp. 386-391. <u>https://tinyurl.com/stablematchings</u>

Roommate matching cards (PDF): <u>http://tiny.cc/my3zuy</u>

Computer simulation: <u>http://www.tiny.cc/roommatematch</u>

For links to this article's resources and more, visit us at <u>www.mathteacherscircle.org/newsletter</u>.

### Case Studies of MTC Participants Published in The Mathematics Educator

*The Mathematics Educator* (TME) recently published an article entitled "The Impact of Math Teachers' Circle Participation: Case Studies," by Brianna Donaldson, Michael Nakamaye, Kristin Umland, and Diana White. The article analyses the self-reported experiences of four MTC participants with varying professional and educational backgrounds over the course of two years. According to the article, "Taken together, evidence from these case studies suggests that for some teachers, participating in a MTC can serve as a catalyst for changing their reported views of mathematics, classroom practice, level of professional engagement, or even all three. ... Providing additional support for the development of productive teacher mindsets about mathematics, connecting with other PD programs that focus on pedagogical skills such as inquiry-oriented teaching, and incorporating opportunities for professional engagement are ways in which MTCs, and perhaps other contentfocused PD models, may be able to support teachers more effectively in their professional growth." The research was supported by a grant from the National Science Foundation's Discovery Research K-12 program to the American Institute of Mathematics. The full article may be accessed on the TME website at <u>http://tme.journals.libs.uga.edu/index.php/tme/article/view/378</u>.

### Oklahoma State Department of Education Awarded Federal Grant to Support Native Youth and Implement Math Circles Statewide

The Oklahoma State Department of Education (OSDE) has been awarded a nearly \$4 million federal Native Youth Community Project grant from the U.S. Department of Education that is aimed at improving college and career readiness for American Indian high school students across the state. "Citizens of Oklahoma's 39 tribal nations are an invaluable part of our state's cultural tapestry and make critical investments in our public school system," said State Superintendent Joy Hofmeister. "These funds will allow us to focus more of our efforts on shaping a comprehensive statewide strategy to increase college and career readiness for American Indian students through high school graduation." Led by OSDE's executive director of American Indian education, Julian Guerrero, Jr., the planned activities include the development of a statewide network of MTCs and associated student Math Circles that will support American Indian students and their teachers.

More information about the grant is available at <u>https://kfor.com/2018/10/08/oklahoma-state-department-of-</u> education-awarded-nearly-4m-grant-for-office-of-american-indian-education/.

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## **Dispatches from the Circles**

Local Updates from Across the Country

### Indigenous Communities • 🗮

The Alliance of Indigenous Math Circles (AIMC) had its Founding Board meeting in conjunction with the 2018 American Indian Science Engineering Society (AISES) meeting. The Founding Board included K-12 teachers and administrators, professors, a student, and a non-profit leader. This Board reviewed the organization's efforts over the past two years and outlined a path forward for AIMC including a full Advisory Board and work on formalizing Mission and Vision statements. Further, AIMC Regional Coordinator, Donna Fernandez, as well as four students co-presented at the conference with Directors Tatiana Shubin and Bob Klein. More information is available at <u>https://aimathcircles.org/</u>. ■

- Contributed by Bob Klein

### Georgia • 🍆

Mihaela Munday and Agegnehu Atena, both of Savannah State University, along with Sandra Riley-Howlett (Johnson High School) and Stacey Bragg (Islands High School), engaged a team of 29 K-12 Savannah-Chatham County Public School System (SCCPSS) math teachers at the inaugural Math Teachers' Circle (MTC) workshop on November 10, 2018. SSU's MTC is the first of its kind in southeast Georgia. It provides local opportunities for K-12 mathematics teachers and higher education faculty to engage in mathematical problem solving and dialogue that stimulates a mathematical growth mindset. The workshop is specifically designed for teachers who work primarily with students from groups traditionally underrepresented in mathematics. Contact Mihaela Munday at mundaym@ savannahstate.edu for more information. - Contributed by Mihaela Munday

### Louisiana • 🎩

Middle and high school teachers from Natchitoches and Shreveport visited the Louisiana School for Math, Science, and the Arts (LSMSA) to attend the first of potentially many Math Circle meetings in the area. The event, hosted by Mathematics Professor Judith Covington of Northwestern State University (NSU), builds on and expands the **North Louisiana Math Teachers' Circle** Covington co-founded and directed in Shreveport, which is approximately one hour from Natchitoches. The event was funded by the Noel Foundation in Shreveport, a non-profit dedicated to supporting cultural arts, education, and community. If interested, contact Judith Covington at <u>covingtonj@nsula.edu</u>. **E** *– Contributed by Judith Covington* 



Sandhills Math Teachers' Circle

### North Carolina • 🚄

The newly formed **Sandhills Math Teachers' Circle** in Cumberland County, N.C., had its inaugural meeting on November 17. Over 45 educators were in attendance and plans are already being made for their next gathering in the Spring. The Sandhills MTC is the newest member of the North Carolina Network of Math Teachers' Circles. For more information, contact Brandi Newell at <u>brandinewell@ccs.k12.nc.us</u>.

The North Carolina Network of Math Teachers' Circles is hosting two separate Summer Camps for its members in 2019. The first camp will be from June 30 to July 2 and is open to all members of the Smoky Mountain MTC, Blue Ridge MTC, High Country MTC and new leadership teams interested in starting a Math Teachers' Circle in their area. The second camp will be held July 25 to 27. This camp is open to members of the Sandhills MTC, Charlotte MTC, Triangle MTC, Central MTC, East Carolina MTC, and Wilmington MTC. Both camps will be hosted at the North Carolina Center for the Advancement of Teaching campus in Cullowhee, N.C. These camps are provided with funding received though the NC GlaxoSmithKline Foundation. If you have any questions, please email Nathan Borchelt at **naborchelt@wcu.edu**.

- Contributed by Nathan Borchelt

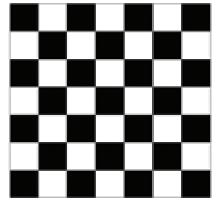
### CONTEST

# Turn Your World On Its Side

### Win a Free Book from MAA AMC

How many ways are there to achieve symmetry in a grid of black and white squares? A scanning code consists of a 7x7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each in this grid of 49 squares. A scanning code is called symmetric if its look does not change when the entire square is rotated by a multiple of 90 degrees counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides. What is the total number of possible symmetric scanning codes?

This problem is contributed by the Mathematical Association of America American Mathematical Competitions (MAA AMC) and first appeared on the 2018 AMC 10A exam. For more information about MAA AMC Competitions, please visit <u>https://www.maa.org/math-competitions</u>.





Send us your solution by February 28, 2019, for a chance to win a free MAA AMC study guide of your choice. Winners can choose between an AMC 8 Math Club Package, AMC 10/12 Math Club Package, a Math Wrangle Handbook, a CD containing past contests and solutions.



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