The Breakout Math Circle

Presented by Kimberly Adams & Donna Farrior
Tulsa Math Teachers Circle - April 2017

But Who's Counting?

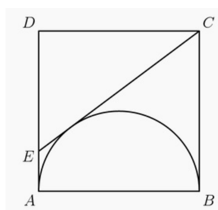
1. Using the letters A, M, O, S, and U, we can form five-letter "words". If these "words" are arranged in alphabetical order, then the "word" $USAMO$ occupies what position?
2. How many ordered pairs (m, n) of positive integers, with $m \geq n$, have the property that their squares differ by 96?
3. How many non-congruent triangles with perimeter 7 have integer side lengths?
4. Six distinct positive integers are randomly chosen between 1 and 2006, inclusive. What is the probability that some pair of these integers has a difference that is a multiple of 5?
5. Suppose we want to order one ice cream sundae with up to 4 servings of toppings. There are 3 possible toppings to choose from. How many different orders of sundaes are possible?

To Open Lock # 3:

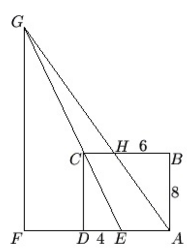
$$(\textbf{\#1 Result} + \textbf{\#5 Result}) + \textbf{\#2 Result}^{(\textbf{\#3 Result}+\textbf{\#4 Result})} = \textbf{Lock Combination}$$

Get In Shape

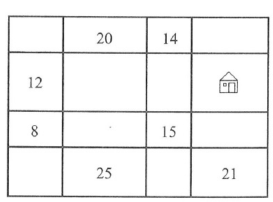
1. Square $ABCD$ has side length 2. A semicircle with diameter $\overline{AB}$ is constructed inside the square, and the tangent to the semicircle from C intersects side $\overline{AD}$ at E. What is the length of $\overline{CE}$?



2. In rectangle $ABCD$, we have $AB = 8, BC = 9$, H is on BC with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G, and F is on line AD with $GF \perp AF$. Find the length of GF.



3. A farm is made up of 15 rectangular fields plus a rectangular acreage on which the farmhouse is built. The acreage of seven of the fields is known, and is shown (but not to scale) below. Determine the acreage of the field on which the house is located.



To Open Lock # 4:

$$\frac{(\textbf{\#2 Result} \times \textbf{\#3 Result})}{\textbf{\#1 Result}} = \textbf{Lock Combination}$$

It's All Logical

1. Determine the five digit number $ABCDE$. Divide $ABCDE$ by 9 and then by 6.

$$\begin{array}{rccccc} & A & B & C & D & E \\ \times & & & & & 4 \\ \hline & E & D & C & B & A \end{array}$$

2. In the puzzle below, each letter stands for a different digit from 0 to 9. The same letter stands for the same digit throughout the problem. Determine the value of each letter.

$$\begin{array}{cccc} \mathrm{O} + \mathrm{W} = \mathrm{D} & \mathrm{D} - \mathrm{G} = \mathrm{R} & \mathrm{W}/\mathrm{O} = \mathrm{G} & \mathrm{C} + \mathrm{M} = \mathrm{C} \\ \mathrm{H} \times \mathrm{A} = \mathrm{H} & \mathrm{O} + \mathrm{O} = \mathrm{W} & \mathrm{O} - \mathrm{G} = \mathrm{A} & \mathrm{H} \times \mathrm{G} = \mathrm{L} \end{array}$$

Sum the digits in the word **WORLD** and divide the result by 3.

3. A marketing company did a survey to find out what kinds of soda people drink. The survey found that 59 drink cola, 43 drink root beer, and 51 drink lemon-lime. Of those, 23 drink cola and root beer, 35 drink cola and lemon-lime, 22 drink root beer and lemon-lime, 15 drink all three kinds of soda and 2 drink none of these three kinds of soda. Determine the following quantity:

$$\frac{\mathrm{A} - 2\,(\mathrm{B} - \mathrm{C})}{\mathrm{D} - \mathrm{E}}$$

where

A:	The total number of people surveyed.
B:	The number of people who do not drink cola.
C:	The number of people who drink root beer but not lemon-lime.
D:	The number of people who drink cola and lemon-lime but not root beer.
E:	The number of people who drink only root beer.

4. Larry tells Mary and Jerry that he is thinking of two consecutive integers from 1 to 10. He tells Mary one of the numbers, and he tells Jerry the other number. Then the following conversation occurs between Mary and Jerry:

Mary:	I don't know your number.
Jerry:	I don't know your number, either.
Mary:	Ah, now I know your number.

Assuming both Mary and Jerry used correct logic, what is the sum of the possible numbers Mary could have?

To Open Lock # 5:

$$\textbf{\#1 Result} + 2\,(\textbf{\#2 Result} + \textbf{\#3 Result} + \textbf{\#4 Result}) = \textbf{3-Digit Number}$$

Replace the middle digit with a letter using the scheme $A = 0, B = 1, \ldots$.

What's Your Theory?

1. Find the digits denoted a and b in $23! = 2585201ab38884976640000$. Form the two-digit number ab.

2. You are in the possession of two bottles, one with a capacity of 7 gallons and one with a capacity of 11 gallons. Next to you is an infinitely large tub of water. You need to measure exactly 2 gallons in one of the bottles. You are only allowed to entirely empty or fill the bottles. You can't fill them partially since there is no indication on the bottles saying how much liquid is in them. How do you measure exactly 2 gallons? Let n represent the number of times you fill a container from the infinitely large tub of water and let k represent the number of times you poured liquid onto the ground. (Neither of these include the transfer of water between bottles.) Determine $n \times k$.

3. Alice and Bill are beginning a month-long bicycle trip when they notice that Alice's trip odometer is broken. At each mile, Alice's odometer advances the digit in the same place as the digit that Bill's odometer advances, but the digits to the right of that digit (if any) do not advance. Thus, the odometer readings for the first 25 miles of their trip are:

Alice's odometer	Bill's odometer
001	001
002	002
003	003
004	004
005	005
006	006
007	007
008	008
009	009
019	010
010	011
011	012
012	013
013	014
014	015
015	016
016	017
017	018
018	019
028	020
029	021
020	022
021	023
022	024
023	025

Bill tells Alice at the end of the trip they have traveled 374 miles. What does Alice's odometer read?

To Open Lock # 6:

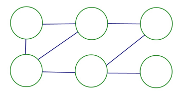
#1 Result + #2 Result + #3 Result = Lock Combination

A Little Bit Puzzling

1. Place $0, 1, 2, 3, 4, 5$ in the circles so that

$$0 \to 4, \quad 1 \to 12, \quad 2 \to 7, \quad 3 \to 8, \quad 4 \to 3, \quad \text{and} \quad 5 \to 4.$$

   By $a \to b$, we mean that the circles connected to a have numbers which add to b.



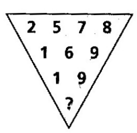
   Each row of the resulting puzzle forms a three-digit number. Determine the difference in the two numbers.

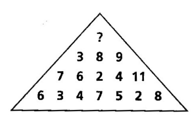
2. The following puzzle is a Binary Puzzle. Each box should contain a zero or a one. No more than two similar numbers next to or below each other are allowed. Each row and each column should contain an equal number of zeros and ones. Each row is unique and each column is unique.

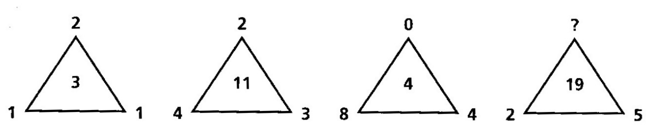
				0	0
	1				
0	1		1		
0				0	
		1			

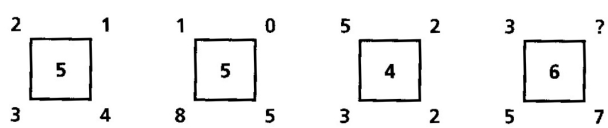
   Once the puzzle is complete, let each row in the puzzle represent a binary number. Convert these binary numbers to base 10 and sum the results.

3. Form a four-digit number using the missing elements(denoted with a ?) in each of the following sequences:









To Open Lock # 1:

$$(\textbf{\#1 Result} + \textbf{\#3 Result}) - \textbf{\#2 Result} = \textbf{Lock Combination}$$

Hopping Around

1. Dan entered the numbers 1 to 9 in the cells of a 3×3 table. He began by placing the numbers $1, 2, 3$ and 4 as shown in the picture. After he was finished, the sum of the numbers in the cells adjacent to (having a common side with) the cell with the number 5 is equal to 9. What is the sum of the numbers in the cells adjacent to the number 6?

1		3
2		4

2. A bicyclist rode the distance of 84 km at a constant speed. Each hour, he rode 2 km farther than he planned, which shortened his whole ride by one hour. What was the bicyclist's speed?

3. Each of eight identical envelopes contains one of the following numbers: 1, 2, 4, 8, 16, 32, 64, 128. Eve chooses a few envelopes randomly. Allie takes the rest. Both sum up their numbers. Eve's sum is 31 more than Allie's. How many envelopes did Allie take?

4. In the Kangaroo republic each month consists of 40 days, numbered 1 to 40. Any day whose number is divisible by 6 is a holiday, and any day whose number is a prime is a holiday. How many times in a month does a single working day occur between two holidays?

To Open Lock # 2:

- Determine the **mod**4 value for each result.
- Determine the direction arrow for each number using the following key:

$$0 \uparrow \qquad 1 \rightarrow \qquad 2 \downarrow \qquad 3 \leftarrow$$

- Use the direction arrows IN ORDER to open the lock.

The Final Set

1. Let $S = 1! + 2! + 3! + 4! + 5! + \ldots + 98! + 99!$. What is the units' digit value of S?

2. Bella is an avid reader. She bought a copy of the bestseller *Math is Beautiful*. On the first day, Bella read 1/5 of the pages plus 12 more. On the second day, she read 1/4 of the remaining pages plus 15 pages. On the third day, she read 1/3 of the remaining pages plus 18 pages. She then realized that there were only 62 pages left to read, which she read the next day. How many pages are in this book?

Set the dials to the following numbers and push the button to the right:

$$(\textbf{\#1 Result} + \textbf{\#2 Result}) = \textbf{Lock Combination}$$