The Mad Veterinarian on Mathematical Safari

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There is a collection of problems which have come to be known as "Mad Veterinarian Puzzles", for reasons which will soon become obvious.

As a warmup, we will look at the "usual" questions which are asked in this context Then we will ask some new types of questions.   The goal is to see just how much structure is contained in these ideas, and really how beautiful, these ideas can be!

If we have some time at the end, we will mention how these puzzles arise in some current mathematical research areas.

Puzzle 1:

Here's your first example of a Mad Veterinarian Puzzle.   (The wording is taken from the most famous Mad Vet Puzzle site, http://www.bumblebeagle.org/madvet/index.html   The details of this particular Puzzle are slightly different than the details of the "Main Mad Vet" Puzzle given at that site.)

A Mad Veterinarian has created three animal transmogrifying machines.

Place a cat in the input bin of the first machine, press the button, and .... whirrr .... bing!   Open the output bins to find four dogs and one mouse.   The second machine can convert a dog into three cats and three mice, and the third machine can convert a mouse into a cat and a dog. Each machine can also operate in reverse (e.g., if you've got two dogs and five mice, you can convert them into a cat.)

This Mad Vet has one cat.

Question 1.1:
  Can he convert it into seven mice?   (If so, show how. If not, explain why not.)

Question 1.2:
  Can he convert it into a pack of dogs, with no cats or mice left over?   (If so, show how. If not, explain why not.)

Puzzle 2:

Another Mad Vet has these three machines:

Machine #1 turns one cat into one dog.
Machine #2 turns one dog into one cat, one dog, and one mouse
Machine #3 turns one mouse into one cat and one dog

This Mad Vet has one cat.

Question 2.1:
  Can she convert it into two cats? (If so, show how. If not, explain why not.)

Question 2.2
 Can she convert it into three cats?   (If so, show how. If not, explain why not.)

Question 2.3:
  Can she convert it into three cats and one dog?   (If so, show how. If not, explain why not.)

Question 2.4:
  Can she convert it into one mouse?   (If so, show how. If not, explain why not.)

Question 2.5:
 Using these three Mad Vet machines, how many different "classes" of animals are there?   Describe all of the classes.

Question 2.6
A friend of this Mad Vet brings her, as a birthday present, a cat and a mouse.   But our Mad Vet prefers to have just one cat.   Can she transmogrify her new collection of animals back to the way things were when she only had one cat?

Question 2.7
Does you answer to the previous question depend on the fact that our Mad Vet started with one cat?   That is, if she started with ANY collection of animals, and her friend brought her a cat and a mouse, would she be able to transmogrify her collection back to the original collection?

Question 2.8

(See Question 2.4)   The Mad Vet is looking to close down the business, but she'd like to finish his Mad Vet business owning only one mouse (and no other animals). From your answer to Question 2.4, you know she can't simply use the cat she has to produce one mouse.   So she'll go to the pet shop down the street and buy some animals, and then use those animals together with her one cat to produce a mouse. What animals should she buy at the pet shop?

Question 2.9

  By adding animals from the pet shop to the cat she currently owns, is it possible for the Mad Vet to finish her Mad Vet business owning whatever collection of animals she'd like?

Question 2.10

  Does your answer to Question 2.9 depend on the fact that she happens to currently own one cat?

Puzzle 3

Yet another Mad Vet has these three machines:

Machine #1 turns one cat into one dog.and one mouse
Machine #2 turns one dog into one cat and one dog
Machine #3 turns one mouse into one cat and one mouse

This Mad Vet has one cat.

Question 3.1:
  Can she convert it into two cats?   Three cats?   Any number of cats?   (If so, show how. If not, explain why not.)

Question 3.2
 Can she convert it into one dog?   (If so, show how. If not, explain why not.)

Question 3.3:
  Can she convert it into one mouse?   (If so, show how. If not, explain why not.)

Question 3.4:
 Using these three Mad Vet machines, how many different "classes" of animals are there?   Describe all of the classes.

Question 3.5
A friend of this Mad Vet insists on giving the Mad Vet a gift of animals for her birthday. Our Mad Vet really prefers to have one cat. So as not to offend her friend, what collection of animals should the Mad Vet tell her friend to bring her for a birthday present?

Question 3.6
Does you answer to the previous question depend on the fact that our Mad Vet started with one cat?   That is, if she started with ANY collection of animals, and her friend brought her a cat and a mouse, would she be able to transmogrify her collection back to the original collection?

Question 3.7
 The Mad Vet is looking to close down the business, but she'd like to finish his Mad Vet business owning only one mouse (and no other animals).   From a previous question, you know she can't simply use the cat she has to produce one mouse.   So

she'll go to the pet shop down the street and buy some animals, and then use those animals together with her one cat to produce a mouse.   What animals should she buy at the pet shop?

Question 3.8

  By adding animals from the pet shop to the cat she currently owns, is it possible for the Mad Vet to finish her Mad Vet business owning whatever collection of animals she'd like?

Question 3.9

  Does your answer to the previous question depend on the fact that she happens to currently own one cat?

Puzzle 4

Our final Mad Vet has these three machines:

Machine #1 turns one cat into one dog
Machine #2 turns one dog into one cat and one mouse
Machine #3 turns one mouse into two mice

This Mad Vet has one cat.

Question 4.1:
  Can he convert it into two cats?   Three cats?   Any number of cats?   (If so, show how. If not, explain why not.)

Question 4.2:
  Can he convert it into one mouse?   (If so, show how. If not, explain why not.)

Question 4.3:
 Using these three Mad Vet machines, how many different "classes" of animals are there?   Describe all of the classes.

Question 4.4
  Is there some collection of animals this Mad Vet should ask to get as a gift from a friend so that he will still be able to have just one cat?

Question 4.5:
 Does your answer to the previous question depend on the fact that the Mad Vet starts with one cat?

Question 4.6:
The Mad Vet is looking to close down the business, but he'd like to finish his Mad Vet business owning only one mouse (and no other animals).   From your answer to a previous question, you know he can't simply use the cat he has to produce one mouse.   So he'll go to the pet shop down the street and buy some animals, and then use those animals together with his one cat to produce a mouse.   What animals should he buy at the pet shop?

Question 4.7
  By adding animals from the pet shop to the cat she currently owns, is it possible for the Mad Vet to finish her Mad Vet business owning whatever collection of animals

she'd like?

Question 4.8

Does your answer to Question 3.6 depend on the fact that she happens to currently own one cat?