# MTCircular

Summer/Autumn 2015

SPECIAL ISSUE

# Putting It All Together

## PERSPECTIVES FROM CIRCLES NATIONWIDE

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Math Teachers' Circle

A NOTE FROM AIM

# Parts of a Whole

Dear Math Teachers' Circle Network,

This special issue of the MTCircular features a broad range of perspectives from Math Teachers' Circles around the country on topics including:

- Supporting teachers mathematically and pedagogically (Michelle Manes, Math Teachers' Circle of Hawai'i; Michael Nakamaye, Albuquerque MTC; James Tanton; and Joshua Taton and Cathryn Anderson, Philadelphia Area MTC)
- Sustaining a MTC over the long term (Jane Long, East Texas MTC; and Fawn Nguyen, MTC of Thousand Oaks, CA)
- Developing statewide networks of MTCs (Angie Hodge, Michelle Homp, and Jim Lewis, Nebraska MTC Network; and Bob Klein, Ohio MTC Network)

Most of these articles stem from presentations during a session on "Math Teachers' Circles and the K-20 Continuum," held at the 2015 Joint Mathematics Meetings and organized by Brian Conrey, Michael Nakamaye, Kristin Umland, and Diana White. This issue's Problem Circle also came from James Tanton's talk at the session, and we are excited to provide a prize donated by Princeton University Press. The lovely "Math Without Words" puzzle was also contributed by James, with an elegant redesign by mathematical artist Natalya St. Clair.

In addition to the statewide networks in Nebraska and Ohio described in the special session articles, several other nascent regional networks of MTCs are beginning to provide a forum for interacting and sharing resources among local groups. A special congratulations to the Smoky Mountain MTC at Western Carolina University, which was recently awarded a multi-year grant from the GlaxoSmithKline Foundation to foster the development of a MTC network in North Carolina.

Finally, I'd like to draw your attention to two new resources on the MTC Network website: the Video Library, with annotated playlists of classic sessions (**http://www.mathteacherscircle.org/resources/video-library/**), and the Organizer Toolkits, with resources for beginning and sustaining your MTC (**http://www.mathteacherscircle.org/resources/toolkits/**).

Happy problem solving!

Brianna Donaldson

Brianna Donaldson, Director of Special Projects

American Institute of Mathematics

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Cover Photo: Nick Benjaminsz

# Putting It All Together

## PERSPECTIVES FROM CIRCLES NATIONWIDE

The 2015 Joint Mathematics Meetings included a special session, organized by Brian Conrey (AIM), Michael Nakamaye (University of New Mexico), Kristin Umland (University of New Mexico), and Diana White (University of Colorado Denver), that focused on innovative Math Teachers' Circle practices and activities. Read on for some highlights from the presentations, and prepare to be inspired!

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***Author bios, resources, and reference information for the Special Session articles can be found at http://www.mathteacherscircle.org/newsletter.***

Stock Photo: Nick Benjaminsz

# Modeling with Mathematics
## DEVELOPING A COMMON LANGUAGE
### by Michelle Manes

The driving force of the Common Core State Standards for Mathematics are the eight mathematical practice standards (**http://www.corestandards.org/Math/Practice/**), describing what it means to learn and do mathematics. The fourth practice standard is "Model with Mathematics," which says in part: "Mathematically proficient students … are comfortable making assumptions and approximations to simplify a complicated situation, realizing that they may need revision later. They are able to identify important quantities in a mathematical situation and map their relationships using tools such as diagrams, two-way tables, graphs, flowcharts, and formulas…"

However, modeling with mathematics seems to be one of the more confusing practice standards. When we asked teachers in our MTC what it means to "model with mathematics," we got answers like:

- "I model, students do."
- "Modeling allows the teacher to demonstrate how something is done. Students are able to see what process is involved to solve a problem."

*Participants in the Math Teachers' Circle of Hawai'i (MaTCH) engage in a round of 'Ulu Maika before exploring the math involved in inventing a new scoring system.*

- "Modeling is showing your students how to work through a problem/concept step-by-step, breaking it down."

Surely this is what "modeling" means in many education courses. But it is completely different from how most mathematicians think of modeling, which is using mathematics to represent and describe (often real-world) systems or situations. One role of our MTC is to help us bridge these kinds of language barriers between how mathematicians and educators talk.

We designed several sessions over three years to illustrate what mathematicians mean when they talk about "modeling with mathematics." These included:

- A session on bioacoustics led by Alexis Rudd, a graduate student in zoology who studies marine mammals
- A session on mathematical questions inspired by the artist Sue Fuller's "String Composition #337"
- A session led by mathematics graduate student Bianca Thompson based on the traditional Hawai'ian game of 'Ulu Maika
- A session on bee population dynamics

Reflecting on what it means to "model with mathematics" after some of these sessions, teachers wrote:

- "The idea of modeling for math is different from what I initially thought (teacher show, student do). Modeling is abstract."
- "Modeling can be done in multiple ways with actual hands-on manipulatives, as well as drawn diagrams (identification space), and that connection made between the two helps to connect the concept into the student's brain."

This feedback suggests that direct experience with mathematical modeling is helping change how teachers define it. This, of course, is a prerequisite for affecting how modeling is taught in the classroom!

# Hamburgers and High Ceilings

## KNOWLEDGE FOR TEACHING AND LEARNING

### by Joshua A. Taton and Cathryn Anderson

To grow professionally, teachers need consistent, high-quality learning opportunities that stretch both their mathematical and pedagogical muscles. The Philadelphia Area Math Teachers' Circle (PAMTC) takes a research-based approach to support teachers in doing both.

In a provocatively titled piece, "Why Do Americans Stink at Math?", Elizabeth Green (2014) relates a vexing anecdote about Americans' struggles with numerical reasoning. She explains that in the 1980's, the A&W Restaurants wanted to compete with the McDonald's Quarter Pounder, and so they introduced a burger with a third-pound of beef at a comparable price. On taste tests, the A&W burger won, hands-down, but failed to sell. A&W eventually learned why: Americans thought that the Quarter Pounder was a better deal, because—you guessed it—they incorrectly reasoned that 1/4 was larger than 1⁄3.

Concerns about Americans' math ability are nothing new. According to Green, these concerns persist, despite changes in educational policies, curriculum, and technologies, largely because our style of teaching has remained stagnant. In particular, as Jo Boaler (2013) explains, American teaching is embedded within a set of faulty beliefs about who can do math and how math is learned, such as the flawed idea that mathematical dexterity requires speed. Another popular misconception, Green notes, is the idea that simply improving teachers' knowledge of math results in improved math teaching. Decades of research, however, have proven otherwise.

### Mathematical Knowledge for Teaching

So, what is it about our style of teaching that needs to change? Insight comes from a series of studies by Deborah Ball and Hyman Bass (2003; see also Ball, Hill, & Bass, 2005), who demonstrated that professional mathematicians and math teachers think differently. This should be obvious to anyone who has struggled to understand an otherwise brilliant math professor. Effective math teaching requires not only knowledge of math content, but also how to communicate—to show useful representations, to anticipate and decipher students' errors, and so on.

Ball and Bass call this broader knowledge "mathematical knowledge for teaching" (MKT). An example of MKT in action is explaining why some students obtain incorrect products such as 245 or 1,055 when trying to multiply 35 and 25. Mathematics professors have a difficult time explaining these errors, perhaps due to their flexible understanding of number systems. On the other hand, experienced math teachers are able to explain them easily, drawing on both strong numerical reasoning and a well-developed understanding of the relationships between concepts and algorithms.

### Building Mathematical Knowledge for Teaching... and our Circle

According to Helen Doerr and her colleagues (Doerr, Goldsmith, Lewis, 2010), high-quality professional learning opportunities involve a number of factors, such as: teachers' engagement in ongoing, collaborative activities; frequent opportunities to notice, analyze, document, and respond to students' thinking; and developing key mathematical and pedagogical habits of mind. As an example of habits of mind, effective teachers manage classroom discussions in ways that involve students and support their reasoning (Chapin, O'Conner, & Anderson, 2003). The key here is that professional learning opportunities must engage both mathematical and pedagogical thinking. Doing one over the other, is akin to teaching cooking by focusing on either ingredients or techniques—without recognizing the need for integrating the two.

In the Philadelphia Area Math Teachers' Circle (PAMTC), we draw heavily on these research-based recommendations for high-quality professional learning. Sure, our workshops involve working together on challenging math problems, of the sort that Jo Boaler (2014) has called "low-floor and high-ceiling" problems. But we go beyond just doing math, because our teachers need—and, importantly, have asked for—tools and frameworks for undertaking problem solving with their students. Our workshops, then, involve discussions not only of mathematical concepts, but also of how the facilitators engaged with participants—by carefully setting up problems, asking leading questions, not "telling" answers, and validating multiple approaches and solutions.

We also review curriculum materials brought in by our teachers, and we investigate research-based ways of increasing the "cognitive demand" (or level of rigor) of typical, procedure-based math problems, which do not allow for deeper thinking about mathematics (Smith & Stein, 1998). Our teachers also suggest ways of incorporating more students' voices and increasing conceptual grappling in their classrooms. For example, one teacher explained that to address a common mistake with the order of operations, he challenges students to communicate their multi-step calculation processes in a single line of writing. This allows them to reason authentically and to problem-solve, rather than to follow the rules blindly.

## Our Teachers' Perspectives

Our teachers often say that finding valuable professional learning opportunities is not easy. They want to be engaged as professionals, rather than simply talked at, and they indicate a strong desire to expand their knowledge of mathematics while also finding (and practicing) innovative ways to further students' knowledge. Traditional professional development workshops simply do not fit the bill—they don't support the development of mathematical knowledge for teaching. Attending PAMTC meetings, our teachers say, has reinvigorated their love for mathematics and their desire to become better instructors.

This year, for example, a recurrent theme at PAMTC meetings has been "asking questions": How can we, as teachers, ask meaningful questions that deepen students' understanding and allow for more and better peer-to-peer interaction? While solving problems at PAMTC meetings, we encourage teachers to ask questions within their groups, rather than simply tell others what they think. This gives teachers practice at understanding each other's thinking, but also at developing questions that provoke elaboration or redefine theories. Our teachers have reported that, in turn, they have also encouraged their students to ask more meaningful questions of each other. This invariably permits a deeper dive into the mathematics. Therefore, we feel, creating an environment where teachers can explore mathematics — in a way that transparently surfaces key aspects of pedagogy — can help foster both teacher and student development.

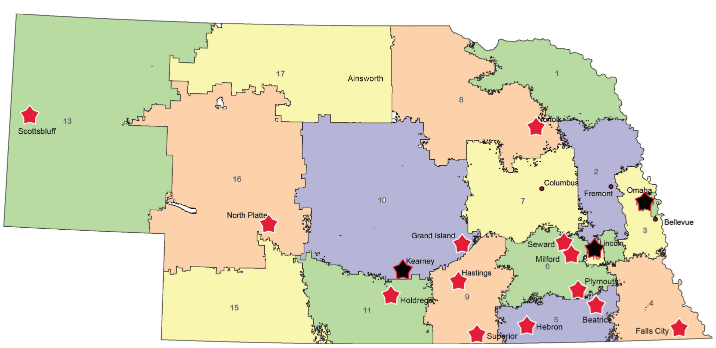
Stock Photo: JCB Spares

*To view references and further reading related to this piece, please visit the resources page of our website at* ***http://www.mathteacherscircle.org/newsletter***.

# Banding Together
## 430 MILES OF MATH IN NEBRASKA
### by Angie Hodge, Michelle Homp and Jim Lewis

There is a rich history of collaboration among K-12 school districts and universities in Nebraska, particularly related to mathematics education. Partnerships formally initiated in the 1990s through a Statewide Systemic Initiative have played a key role in professional development and student achievement for more than a decade, and have led to the acquisition of several NSF grants, including Math in the Middle, a $5.9 million Math and Science Partnership grant (2004-2009), and NebraskaMATH, a $9.2 million grant (2009-2014), which developed a strong professional network of K-12 teacher-leaders and university faculty.

Math Teachers' Circles are a natural extension of these partnerships. The longest running Circle in Nebraska, and one of the longest running Circles in the nation, is the Lincoln Area Math Teachers' Circle. This well-attended Circle was formed by a team of five individuals, including two K-12 teacher graduates of the Math in the Middle program, who attended the first national "How to Run a Math Teachers' Circle" workshop in 2007. The impact of the Lincoln Area MTC has extended across the state and even beyond Nebraska as UNL graduate students have gone on to form MTCs in their respective institutions after graduation. Within Nebraska, Pari Ford at the University of Nebraska at Kearney (a UNL Ph.D. alumna) formed the Central Nebraska Math Teachers Circle in 2011. In addition, connections with neighboring UNO also led to the initiation of the Omaha Area Math Teachers' Circle in 2010. Teachers are recruited heavily from the NebraskaMATH program, as these teachers have already developed relationships with the MTC leaders.




*Locations of Nebraska MTCs in Lincoln, Omaha and Kearney are shown as black stars; locations of GNMTC events are shown in red.*

Though initially focused in Lincoln, Omaha, and Kearney, the three most densely populated areas in Nebraska, MTCs are now impacting teachers 430 miles across the state, thanks to the Greater Nebraska Math Teachers' Circle (GNMTC). The GNMTC draws on our existing statewide partnerships to extend the benefits of Math Teachers' Circles to teachers in the rural locations that comprise the vast majority (geographically) of the state.

Rather than having several meetings in a single location, the GNMTC supports a single meeting in several locations. Each academic year begins with a dinner meeting in a central Nebraska location, attended by individuals from about 15 rural districts who will coordinate meetings for grades 6-12 teachers in their home communities. The meeting is held the evening before the Fall Conference of Nebraska's state NCTM affiliate, the Nebraska Association of Teachers of Mathematics (NATM), which draws more than 400 K-12 teachers each year. Teachers participate in a well-planned mathematical activity designed by a K-12 teacher leader and a UNL mathematics graduate student to be challenging for GNMTC teachers and also potentially adaptable to teachers' own classrooms. Lesson plans, slides, and handouts are distributed to those in attendance. This initial event also enables us to recruit teachers to lead the activity at a Circle in their respective locations, with the promise that GNMTC coordinators will help with advertising and provide funding for food costs.

Holding a single meeting in several locations has been a successful, sustainable model for the GNMTC. In rural communities, there are too few teachers to schedule meetings frequently in any one location. These teachers are often taking on multiple roles at their schools. For example, many are also athletic coaches, and so we have found that Circles scheduled in early November generally are the most successful, largely because this time period falls between the fall and the winter sports seasons. A comment from a rural teacher on a Circle evaluation suggests that we are achieving our goal of benefiting teachers in isolated areas: "Connecting with others is always beneficial to my teaching, I learn something from each of them. In a small district, communicating with other math teachers is difficult. This provides a chance to do so."

# JOINING FORCES IN OHIO

## by Bob Klein

In November 2014, the Cincy MTC and SEOMTC joined together at the Ohio Council of Teachers of Mathematics state conference to present two MTC workshops. Engaging participants in Rational Tangles and Liar's Bingo, these groups generated a buzz about MTCs and their potential as content-based professional development.

Following up on this excitement, on April 24-25 this year, a group of Ohio Math Teachers' Circle facilitators gathered at an MTC summit to discuss the formation of a regional network of MTCs. Representatives from the Black Swamp Math Teachers' Circle (Toledo/Bowling Green), Cincinnati Math Teachers' Circle, Columbus Math Teachers' Circle, and SouthEast Ohio Math Teachers' Circle (Athens) met for two days in Columbus. They were joined by a group of educators and higher education faculty from Cleveland and Youngstown who expressed interest in starting new MTCs in those cities.

To capitalize on the energy and the perceived "critical mass" of MTCs in Ohio, the MTC Network in Ohio hopes to form a loose confederation of MTCs to afford the possibilities of:

- Creating a website that details MTCs in the state and their connections to state-specific initiatives;
- Sharing funding opportunities, especially around summer immersion workshops;
- Maintaining a directory of MTC presenters and topics to share with other MTCs in the state;
- Nurturing new MTCs by providing support and local encouragement as well as "proof-of-concept" models; and
- Continuing conversations about sustainability of MTCs and the challenges that all MTCs face nationally.

While the MTCs in Ohio differ significantly in frequency of meetings, target populations, summer immersion planning, and other ways, the summit allowed existing MTCs to explore these differences and commonalities and to establish a community around our common purpose. Moreover, the Cleveland and Youngstown area groups are actively working on forming MTCs in their area with the support of this new network. The network aims to be strong enough to foster mutual support yet loose enough to allow each different group to flourish.

*Ohio MTC Summit participants build polyhedra during a math session.*

# A Circle of Friends

## UNLIKELY PARTNERSHIPS FOR SUCCESS

### by Jane Long

When I first learned about Math Teachers' Circles, I started to think of them as a group of friends working together on problems. I've found this to be true in more than just the mathematical sense as I led the East Texas Circle. Our Circle is now finishing its second successful year, largely due to an unexpected friendship between organizations.

After hearing about the North Louisiana Circle nearby, some colleagues and I attended AIM's "How to Run a Math Teachers' Circle" workshop and planned to build off of our connections with area teachers from earlier grant programs. We returned with enthusiasm, applied for seed grants from MSRI and AIM and arranged to hold our meetings at Stephen F. Austin State University's STEM Center. But our big push came when we partnered with GEAR UP, a federal grant program aimed at improving college readiness partly by enriching teachers. The program was already established on our campus, and they needed professional development for their math teachers. The program paid for meals, presenters, books, and, most importantly, paid teachers to attend. GEAR UP has supported us through two summer workshops and eight meetings per year, and their partnership also prompted us to invest in program evaluation. This was an excellent beginning!

We found this partnership because we were looking for opportunities. When starting something new, it's often the case that existing models and funding sources don't seem to fit our needs and expectations exactly. That's just another opportunity to problem solve! I have learned to look for programs or grants that share a similar mission, then ask them for what I really want. Some restrictions are not flexible, but some are. For example, we made sure that our Circle had an "open door" policy, so that teachers not involved in the GEAR UP program could still attend. We paid for their meals and supplies through our seed grants and encouraged them to bring friends and colleagues. Assuming too much about an opportunity's limits may cause you to miss out on a powerful partnership.

Our Math Teachers' Circle is now an established part of our department's culture and word about our offerings is spreading among area teachers. I have high hopes that we will be able to continue this momentum when our partnership with GEAR UP ends this summer. We are already working on some new friendships and partnerships, and I can't wait to see the new directions our Circle will take.

*The East Texas Math Teachers' Circle takes a break from problem solving.*

# Plan, Deliver, Build Community

## THREE KEYS TO SUSTAINABILITY

### by Fawn Nguyen

Back in January, at the Joint Mathematics Meeting in San Antonio, I was privileged to talk about our Math Teachers' Circle in beautiful Thousand Oaks, California. I presented three key components that help us sustain a viable community of math teachers and math professors.

The three key components are: 1) Planning, 2) Delivering, and 3) Building Community. We invested time and resources in these components when we first established our Circle three years ago. Since then, we have continued to contribute equal attention to these key structures as we work to sustain the success of our Circle.

Planning involves applying for grants, scheduling speakers, and getting the word out to math teachers. We currently have secured three grants, one of which is directly from our host university – a testament to the institution's unwavering support to keep and grow our Circle. We use the funds for speakers' honoraria and catered dinners. Scheduling speakers means we need to contact people months in advance. We need to have our dates set, yet be flexible enough to change them to meet our speaker's timeline.

Another critical part of planning is advertising. Our website (**http://www.mathteacherscircleto.org/**) is one way we advertise, but we also send out an electronic flyer to past participants and to the math specialist of our county. Our professors invite student teachers at the university, and I invite teachers from the workshops where I present.

The second key component is delivering. We are only as good as what we can deliver to our attendees. We make sure we present rich mathematical problems that are non-routine and have various solution strategies. Our goal is for teachers to take on the role of the learner while enriching their own mathematical experience. We emphasize the appropriate Common Core State Standards for Mathematical Practice in every session. At a minimum we want teachers to struggle productively (persevere!) and critique each other's reasoning at every session, no matter what the problem is. It is not the goal of our delivery for teachers to repeat the same task with their students, but rather for teachers to come away inspired and confident to teach mathematics in ways that foster critical thinking and collaboration.

Building Community means taking care of our people — a professional group of math professors, teachers, and future teachers — so that folks will come back and invite others to join! We value our attendees by serving a delicious dinner at the start of each session. This is our time for conversations, to meet someone new, or to catch up with a colleague. We value our attendees by starting on time and finishing on time. We provide attendees with resources related to each session. We understand that the building of our Math Teachers' Circle community starts and ends at respecting our teachers and the very hard work that they do as educators.

*The Math Teachers' Circle of Thousand Oaks works on a knotted problem together.*

# One, Two, Three, Four

## BUILDING NUMBERS WITH FOUR OPERATIONS

### by Michael Nakamaye

In a recent session of the Albuquerque MTC, we investigated a series of questions about building whole numbers out of 1, 2, 3, and 4, using addition, subtraction, multiplication, and division. For example, we can write 11 = (2 x 3) + (4 + 1).

Using parentheses is natural and important in order to indicate the order in which operations take place. In the expression above, we get a very different result by changing the parentheses: 15 = (2 x (3 + 4)) + 1.

One interesting question is to examine how many different results we can obtain by changing the location of the parentheses.

Moving in another direction, how many different ways are there to make 11? Here we will allow expressions that do not use all of the numbers 1, 2, 3, and 4. So, for example, (3 x 4) - 1 is a second way to make 11. A third way to find 11 is 11 = ((4 - 1) x 3) + 2.

Are there other ways? Investigating this problem will require patience, creativity, and persistence. Other interesting questions include:

- Which number can be made in the most different ways?
- What is the smallest whole number that can only be made in one way?
- What is the smallest whole number that cannot be made?

One trend that emerges is that to get the "larger" numbers, multiplication is more efficient than addition. As the target number increases, a good strategy is often to look at its prime factorization. For example, 21 = 3 x 7, and so we can write 21 = 3 x (1 + 2 + 4) or 21 = 3 x ((2 x 4) - 1). Similarly, 22 = 2 x 11, and this can lead to the expression 22 = 2 x ((3 x 4) - 1).

Related to this observation, prime numbers arise naturally in investigating this problem, since they cannot be factored as a product of two smaller numbers. For example, there is no way to build 19 as a product, other than 19 and 1. This means that the last operation used to reach 19 will not be multiplication but rather addition or subtraction. (The last operation will not be division because we cannot reach a multiple of 19 using only three of the numbers.) One way to write 19 is 19 = (4 x (2 + 3)) – 1, and, sure enough, the final operation is subtraction.

Natural extensions of this problem quickly lead into the unknown. Suppose we use 1, 2, 3, 4, and 5. What is the smallest whole number that we cannot make? The author does not know the answer to this question and started to struggle pretty mightily once reaching some of the numbers in the 70s: for example 74 = ((3 x 4) x (5 + 1)) + 2.

A few more questions include:

- What is the largest positive (or smallest negative) number that you can make?
- Does removing division change the answers to any of the problems? What if we allow additional operations such as exponentiation?
- If $f(n)$ is the smallest number that cannot be made with 1,…, $n$, what can we say about $f(n)$?
  - How often is $f(n)$ prime?
  - How does $f(n)$ grow relative to the size of $n$?
  - How is $f(n)$ related to $f(n - 1)$? For example, can we estimate the size of $(f(n+1))/(f(n))$?

Work on these problems builds arithmetic fluency and provides opportunities to identify patterns, develop and defend arguments, and create conjectures. This investigation also highlights how thin the boundary is between a fun warm-up activity for fifth graders practicing their arithmetic and deep questions investigated by research mathematicians!

# Dispatches from the Circles

### Local Updates from Across the Country

## Illinois •

The **Great Rivers Math Teachers' Circle** (located in Edwardsville, Illinois, near St. Louis) received a $2880 grant from the Meridian Society, a group of women philanthropists that support projects that partner community organizations with the university. Our Circle started in Spring 2015 on a shoestring, but this funding, combined with support from AIM, positions us to have a strong group of middle school teachers for the 2015-16 academic year.

– *Contributed by Adam Weyhaupt*

## Michigan •

The **Wayne County Math Teachers' Circle** was recently recognized by the Detroit Area Council of Teachers of Mathematics (DACTM) with a $500 gift and an award presented to Nina White.

– *Contributed by Nina White*

## Mississippi •

The **Mississippi Delta MTC** will hold an immersion workshop in late July. The leadership team will also be working together throughout June on our MSP grant, providing 4 weeks of professional development to middle school math teachers.

Co-Director Liza Cope received a National Science Foundation early-career travel scholarship to attend a four-day summer Inquiry-Based Learning workshop at California Polytechnical Institute in July 2015.

Cope was also just elected "Member-at-Large" at the annual symposium of the Mississippi Association of Mathematics Teacher Educators.

For more information about our upcoming activities, please see this recent article released by Delta State University: http://www.deltastate.edu/news-and-events/group-continues-math-teachers-circle-planning/.

– *Contributed by Liza Cope*

## Interested in Starting a Circle?

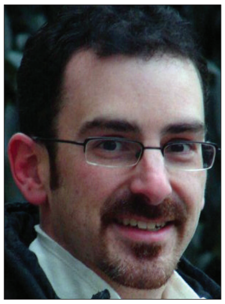
The Math Teachers' Circle Network provides support to help get you started! We offer:

- Seed funding
- A visit to a nearby Math Teachers' Circle
- A Circle Mentor
- Planning assistance

Learn more at http://www.mathteacherscircle.org/start-a-circle/ or contact us via email at circles@aimath.org.

# Bolognese, Taton Publish MTC-Related Articles


*Bolognese*


*Taton*

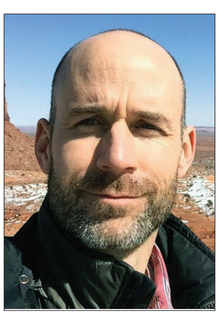
"Brownie Points," by Chris Bolognese (Columbus Academy Upper School and Columbus MTC), was recently published in the North American GeoGebra Journal (**http://www.geogebrajournal.com/index.php/ggbj/article/view/46/43**). In the article, Bolognese describes a GeoGebra-aided exploration of a favorite MTC problem that involves fairly dividing a pan of brownies with a missing piece.

"Much More Than It's Cooked-Up To Be: Reflections on Doing Math and Teachers' Professional Learning," by Joshua Taton (University of Pennsylvania and Philadelphia Area MTC), appeared in the Spring 2015 issue of Perspectives on Urban Education (**http://www.urbanedjournal.org/archive/volume-12-issue-1-spring-2015/much-more-it's-cooked-be-reflections-doing-math-and-teachers'-**). The article argues for the importance of teachers becoming practitioners of the content they teach and describes the role Math Teachers' Circles can play in this process.

# Hagan Named One of NPR's 50 Great Teachers

National Public Radio's "50 Great Teachers" series recently featured Sarah Hagan of Drumright High School and the Tulsa MTC. The 25-year-old teaches at a rural high school using unconventional methods. For example, instead of textbooks, she gives students blank composition books, which they fill with lessons that Hagan writes or open-sources from other teachers across the country. According to the NPR story, "Eventually, the books look like dog-eared, bulging relics from an Indiana Jones movie. Hagan argues that if students are allowed to be creative, they're more likely to remember what they've learned." The full story is available at **http://kosu.org/post/teacher-who-believes-math-equals-love**. Hagan blogs at Math=Love (**http://mathequalslove.blogspot.com**).

# Klein Honored with Presidential Teacher Award



Bob Klein (Ohio University and SouthEast Ohio MTC) was recently named a recipient of Ohio University's Presidential Teacher Award for 2015. The Presidential Teacher Award is based on excellence in teaching and meritorious academic pursuits both inside and outside the classroom, as acknowledged by peers and students, including teaching practices and innovations, influences on curriculum, student mentoring, colleague mentoring, and scholarship with respect to teaching. In addition to his work with the SouthEast Ohio MTC and the developing Ohio MTC Network, Klein is also a co-director of the Navajo Nation Math Circle and the Chair Elect of the MAA's Special Interest Group on Math Circles for Students and Teachers. The award will be presented at a ceremony during Fall 2015.

# Lewis Honored with MAA and AMS Awards

W. James "Jim" Lewis was recently honored with both the MAA Gung and Hu Award for Distinguished Service to Mathematics and the AMS Award for Impact on the Teaching and Learning of Mathematics. The MAA award recognizes extraordinary contributions that have shaped mathematics and mathematical education on a national scale, and the AMS award honors a mathematician who has made significant contributions of lasting value to education in the field. "Throughout his career, Jim Lewis has been a tireless advocate of the idea that mathematics research and education go hand in hand," according to the AMS citation. An Aaron Douglas Professor of Mathematics and the Director of the Center for Science, Mathematics and Computer Education at the University of Nebraska-Lincoln, Lewis is currently serving as Deputy Assistant Director of the Directorate for Education and Human Resources at the National Science Foundation. He is a member of the MTC Advisory Board and has been instrumental in establishing a thriving network of MTCs across Nebraska (see "Banding Together," page 8 of this issue).

# Manes Wins Golden Section Teaching Award

Michelle Manes (University of Hawai'i at Manoa and Math Teachers' Circle of Hawai'i) received the 2015 Golden Section Award for Distinguished College or University Teaching of Mathematics from the Mathematical Association of America (MAA). The MAA's Golden Section includes Northern California, Nevada, and Hawai'i. The citation reads, in part, "An active researcher in the intersection of dynamical systems and number theory, Michelle's teaching style is informed by her grounding in mathematical education research and her conviction that all students are capable of learning mathematics." Manes describes her teaching philosophy as "finding the joy in mathematics, setting high but attainable expectations, communicating to students a belief in their abilities, and maintaining a reflective teaching practice." She was presented with the award during this year's MAA Sectional Meeting at Foothill College in Los Altos, Calif.

# Measuring Up: "Perfect" Rulers

## by Chris Bolognese and Raj Shah

**"Mathematicians enjoy thinking about the simplest possible things, and the simplest possible things are imaginary." — Paul Lockhart**

People who wonder about mathematical objects and ideas see math not as the quest for The Answer, but as an opportunity to play and discover. Humans are wired to think in this manner.

This sense of wonder is what motivates a question regarding a "perfect ruler" which was recently explored at our Columbus Math Teachers' Circle: Is it possible to measure all possible integer lengths on a ruler without marking every integer on that ruler? In particular, can you construct the most efficient ruler that can measure all integer lengths from 1 inch to 36 inches on a yardstick using the least number of marks? And if so, what is the minimum number of marks needed and where should they be placed? A trivial solution would be to mark the ruler at one inch and simply measure objects by moving the ruler along the object one inch at a time. So, we constrain this exploration to using the ruler without moving it along the object.

### Observations

As is often the case, it may be easiest to begin the exploration with a simpler example. Many of our Circle's participants chose to start by analyzing a 6-inch ruler such as the one shown in Figure 1. Take a moment to strategize about how you might make this ruler most efficient.

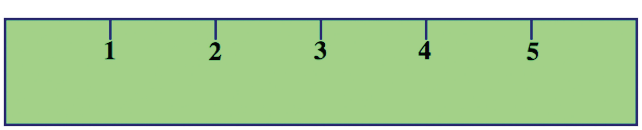



*Fig 1. A standard 6-inch ruler with whole number increments*

Participants primarily used guess-and-check to remove marks at various locations. For instance, the mark at 2 inches could be removed, since an object of length 2 inches can be still be measured, say, as the difference in length between the marks at 1 inch and 3 inches. Additionally, we can remove the mark at 5 inches, since we can still measure a length of 5 inches from the 1-inch mark to the right end. If we remove the mark at 3 inches, we can still measure a length of 3 inches between the 1-inch and 4-inch mark (and a length of 2 inches can still be measured between the 4-inch mark and the right end of the ruler). Since we cannot do even better with just one mark, a most efficient ruler of length 6 inches requires only two marks (Figure 2).


*Fig 2. A most efficient 6-inch ruler with just two marks*

### Conjectures

With a successful exploration in hand for the 6-inch ruler, some groups advanced to longer ruler lengths, while others reverted back to analyze even simpler rulers of shorter lengths to look for a cumulative pattern. After additional time to explore, groups shared their conjectures, including but not limited to the following:

- A most efficient ruler must have at least one mark either 1 inch from the left or right end, in order to measure one less than the ruler's length.
- Symmetry about the midpoint of the ruler allows for twice the amount of possible solutions. For example, marks could be placed at 2 inches and 5 inches, instead of 1 inch and 4 inches, to create another most efficient 6-inch ruler.
- Marks at the Triangular Numbers (1, 3, 6, 10, 15, 21, etc.) might be an efficient marking scheme. So might marks at the Fibonacci Numbers (1, 2, 3, 5, 8, 13, 21, etc.).

- Marks at consecutive integers are not an efficient marking scheme.

## Delving Deeper

While the original task of finding the most efficient yardstick was not exclusively answered by our Circle, a lot of beautiful mathematics unfolded in the process. To investigate this situation further, two digital tools were created after the session by Math Circle members. One tool was written in Javascript to allow the user to choose a length and the location of marks (Figure 3). The tool keeps track of which lengths are measurable and which still cannot be measured. See if you can find any patterns between the length of the ruler and the number of minimum marks required. It is free to use at http://gadgets.mathplusacademy.com/ruler/ruler.html.

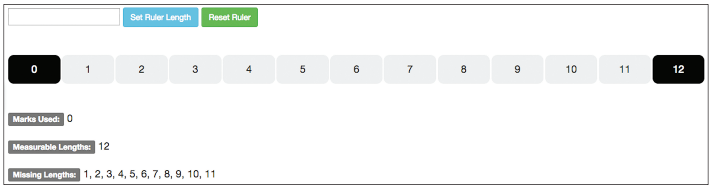



*Fig 3. Applet to dynamically investigate the ruler problem*

Leveraging computer science even farther, we wrote open-source Ruby code to compute the most efficient rulers of various lengths. Visit it online at https://ideone.com/rd0OzG. One interesting result from this code is the minimum number of marks as a function of length, graphed in Figure 4 at above right. Note that this includes marking schemes that are mirror images of each other. We feel that this graph most closely resembles the graph of a square root function. Do you see any other graphical relationships?

## Conclusion

"Perfect rulers" proved to be an engaging and challenging problem for all. It fit the criteria for a "rich mathematical task":

- Easy to understand with a low barrier to entry
- Offers opportunities for initial success to promote engagement
- An "open middle" that allows for creativity and multiple strategies and tools

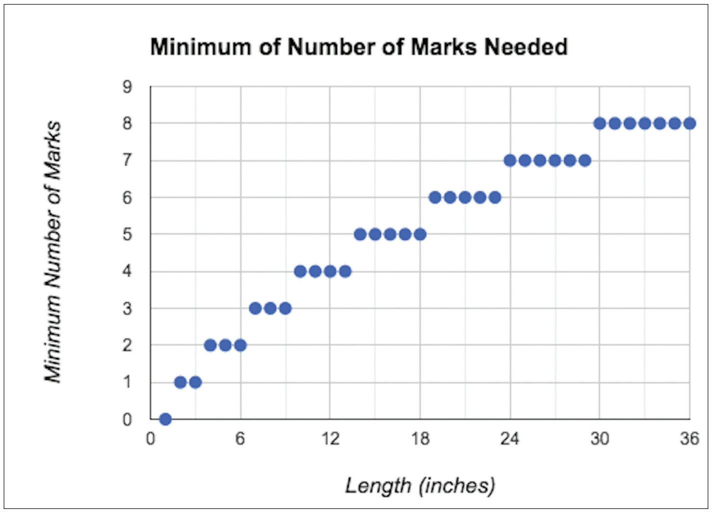



*Fig 4. Plot of the minimum number of marks for a ruler of a given length*

- Encourages collaboration and discussion
- Easily extendable with 'what if' and 'what if not' questions

It is interesting to note that the original "perfect ruler" problem could not be solved by any group during the Circle meeting. It is precisely this fact that led several participants to pursue their investigation of this problem beyond the meeting. It is a reminder to teachers at all levels that it is often beneficial to allow rich mathematical problems to remain unsolved because it inspires mathematical thinking outside the classroom. We encourage your Circle to explore this problem at length!


*Chris Bolognese (**bolognesechris@gmail.com**) is a mathematics teacher and Department Chair for Columbus Academy Upper School and is a co-founder of the Columbus Mathematics Teachers' Circle. Raj Shah (**raj@mathplusacademy.com**) is the founder of Math Plus Academy©, a business for K-8 students to excel in STEM learning, and is a regular participant and presenter at the Columbus Mathematics Teachers' Circle. Find more information about our Circle at **https://columbusmathcircle.wordpress.com/**. Raj was introduced to this problem by Matt Enlow, who teaches Upper School Mathematics at the Dana Hall School in Boston, Mass.*

PROBLEM CIRCLE

# Let Your Digits Do the Multiplying

## by James Tanton

Don't memorize your multiplication tables. Let your fingers do the work!

If you are comfortable with multiples of two, three, four, and five, then there is an easy way to compute the six through ten products in a multiplication table.

First encode numbers this way:

A closed fist represents five and any finger raised on that hand adds one to that value.

Thus, a hand with two fingers raised, for example, represents seven. A hand with three fingers raised represents eight.

To multiply two numbers between five and ten, do the following:

1. Encode the two numbers, one on each hand.
2. Count ten for each raised finger and remember that number.
3. Count the number of unraised fingers in each hand and multiply together those two counts.
4. Add the results of steps two and three. This is the desired product.

For example, "seven times eight" is represented in the figure as two raised fingers on the right hand, three on the left hand. There are five raised fingers in all, yielding the number 50 for step two. The right hand has three lowered fingers and the left has two. We compute: 3 x 2 = 6. Thus the desired product is 50 + 6 = 56.


Similarly, "nine times seven" is computed as six raised fingers plus 1 x 3 giving 63, and "nine times nine" as eight raised fingers plus 1 x 1 giving 81.

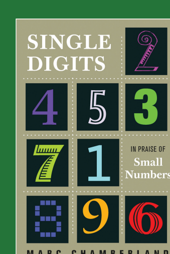
One is never required to multiply two numbers greater than five.

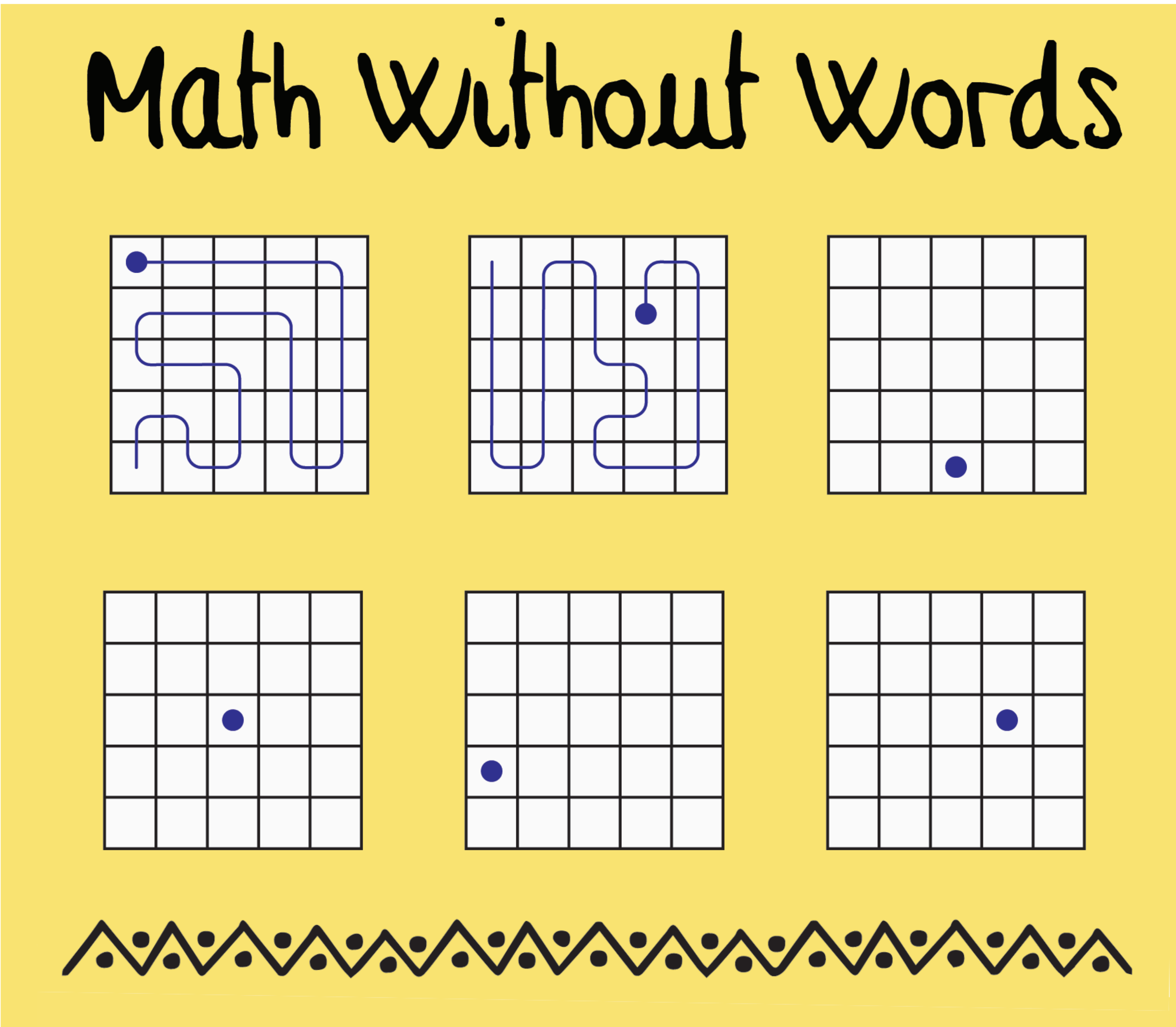
Why does this work?

*For those who want to use their toes, too, or who are curious about extraterrestrial finger multiplication, please check out Tanton's additional challenges at* [http://www.mathteacherscircle.org/newsletter](http://www.mathteacherscircle.org/newsletter).

## WIN A FREE BOOK FROM PRINCETON UNIVERSITY PRESS!

**Send us your solution by September 8 for a chance to win a free copy of *Single Digits: In Praise of Small Numbers* by Marc Chamberland. Through September 30, you can also use code P06096 to receive 30 percent off your purchase of *Single Digits* at press.princeton.edu. Thanks to Princeton University Press for their generosity in donating the prize and extending this discount to our readers!**






*See what paths you can take in this puzzle from James Tanton's "Math Without Words" series. For the story behind the puzzle, check out Tanton's essay "What Made Me a Mathematician" on his website, **http://www.jamestanton.com**. Puzzle copyright James Tanton 2010, image redesigned by mathematical artist Natalya St. Clair (**https://mathartist.wordpress.com**). Reprinted with permission.*

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