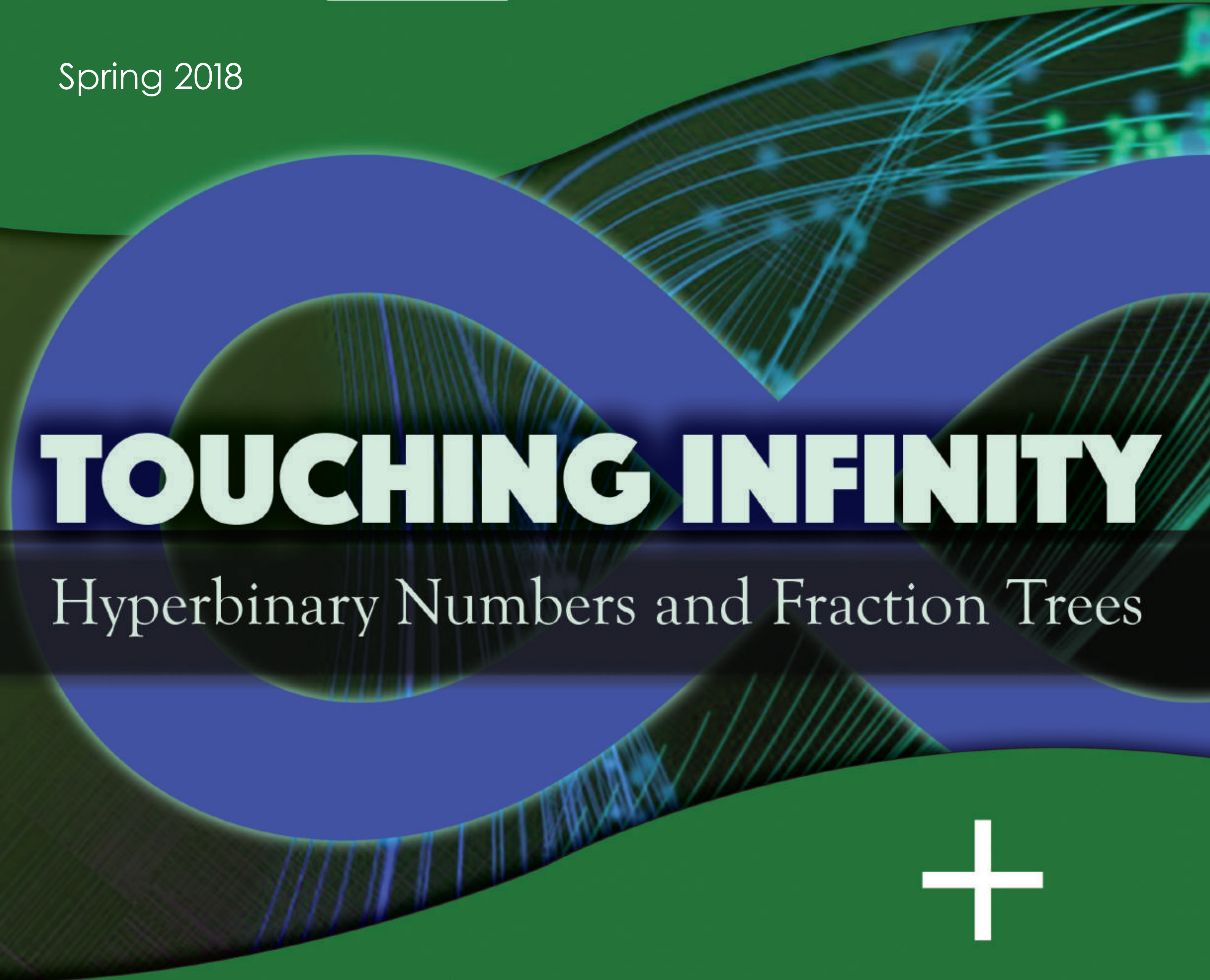
MTCircular

Spring 2018



TOUCHING INFINITY

Hyperbinary Numbers and Fraction Trees

+

Listen, Share, Play Lessons in Problem Solving

Locked Out A Breakout Box Session for Your Circle

Tiling With Pentagons Newly Discovered Math

Global Math Week Exploding Dots from Coast to Coast

Math Teachers' Circle π

A NOTE FROM AIM

"It Had To Start With Me."

Dear Math Teachers' Circle Network,

Happy New Year! If you're looking for topics for your spring MTC sessions, this issue will not disappoint. In "Touching Infinity," Samuel Coskey, Paul Ellis, and Japheth Wood provide a lovely and accessible treatment of questions about infinity. Judith Covington's "Tiling with Pentagons" captures the excitement of investigating brand-new mathematics during a MTC session. Peter Tingley draws inspiration from his preschool-aged daughter to bring a fresh perspective to problem solving in "Listen, Share, Play." And Kimberly Adams and Donna Farrior lead their MTC in an escape room adventure in "Locked Out."

We're proud to share that the MTC Network was recently highlighted as a "bright spot" in teacher professional development by 100Kin10, a national network with the goal of training and retaining 100,000 excellent STEM teachers (**http://100kin10.org**). 100Kin10 also published a "Story from the Field" by the AIM MTC's Heather Danforth-Clayson, describing how her experiences with MTCs have affected her teaching. Heather writes that after five years of MTC participation:

> *My enthusiasm – not just tolerance – for math infects everyone around me. My students feel my excitement, they soak it in, and they believe that they can do math, too. This is an element that was missing when I started teaching. I wanted to find the perfect way to teach so that I wouldn't turn off any students to math. But what I didn't understand at the time is that it had to start with me. (**https://grandchallenges.100kin10.org/progress/heather-danforth-clayson**)*

"It had to start with me." The mathematical conversations, collaborations, and communities we build together in MTCs all fundamentally depend on many individuals making this precise commitment to ourselves, our students, and each other. I want to start my own New Year off by thanking each of you for making this commitment, which inspires me every day.

Happy problem solving!

Brianna Donaldson

Brianna Donaldson, Director of Special Projects

American Institute of Mathematics

The Math Teachers' Circle Network is a project of the American Institute of Mathematics
600 E. Brokaw Road, San Jose, CA 95112
Phone: 408-350-2088
Email: circles@aimath.org
Website: http://www.mathteacherscircle.org/
http://www.facebook.com/mtcnetwork
@mathteachcircle

TABLE OF CONTENTS

Cover photo background by Avery C. at FreeImages.com.

TOUCHING INFINITY

Hyperbinary Numbers and Fraction Trees

BY SAMUEL COSKEY, PAUL ELLIS AND JAPHETH WOOD

Are there more fractions than counting numbers? Surprisingly, an investigation into binary notation can help us answer this question!

Questions about infinity are fascinating, and can lead into deep mathematical topics in set theory. The mathematics of infinite sets wasn't clearly understood until Cantor defined cardinal numbers in the late 19th century, stating that two sets are the same size if there is a one-to-one correspondence between them. One surprising result from set theory, first proved by Cantor in 1873, is that there are *precisely* as many rational numbers (fractions) as there are counting numbers. Over one hundred years later, mathematicians Neil Calkin and Herbert S. Wilf published a more elegant proof of this fact.

This article is the result of our work to develop the ideas in the Calkin-Wilf proof, so that they would be accessible to the teachers in our three different Math Teachers' Circles. We designed an investigation into the hyperbinary numbers (itself a 19th century topic that predates Cantor's work on cardinality) and developed the Tree of Fractions, much in the style of Calkin and Wilf. We asked teachers to make observations, ask questions, and convince each other of the veracity of their claims.

Hyperbinary Numbers

In the *binary* number system, each power of two can be used at most once to represent a positive integer. Every positive integer can be written in binary in precisely one way, and we usually write the sum in order of decreasing powers of two. For example, $27_{10} = 11011_2$, meaning that

$$27 = 16 + 8 + 2 + 1$$
$$= 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 = 11011_2$$

In the *hyperbinary* number system, each power of two can be used at most twice to represent a positive integer. This gives additional ways to represent numbers. For example, a second possible way to represent 27 in hyperbinary is

$$16 + 4 + 4 + 2 + 1 = 10211_2$$

A teacher asked, "How many hyperbinary representations does 27 have?" This is exactly the question we had hoped for! More generally, what are all the ways to represent each counting number in hyperbinary? To investigate, we provided teachers with a collection of manipulatives of various shapes or colors and paper and pencils to tabulate results. One of us used a collection of tiles with different shapes, another used a box of Lucky Charms cereal, and another used 3D-printed binary coins. We agreed that each different object would represent a power of two. For example, if Triangle = 1, Diamond = 2, Square = 4, Trapezoid = 8, and Hexagon = 16, then 27 may be represented using one Hexagon, two Squares, one Diamond, and one Triangle. We also provided a data sheet with one line for each counting number up to 20, available online at https://www.mathteacherscircle.org/newsletter/.

Figure 1 shows the first few lines of data that the participants will produce, written in hyperbinary (participants could also choose to use symbols). In our MTCs, the teachers collaborated to ensure that the representations were correct and complete, and then entered the number of hyperbinary representations of each number into the last column, denoted by $b(n)$.

n	hyperbinary representations of n	$b(n)$
0	0	1
1	1	1
2	10, 2	2
3	11	1
4	100, 20, 12	3
5	101, 21	2
6	110, 102, 22	3
7	111	1
8	1000, 200, 120, 112	4
9	1001, 201, 121	3
10	1010, 210, 1002, 202, 122	5

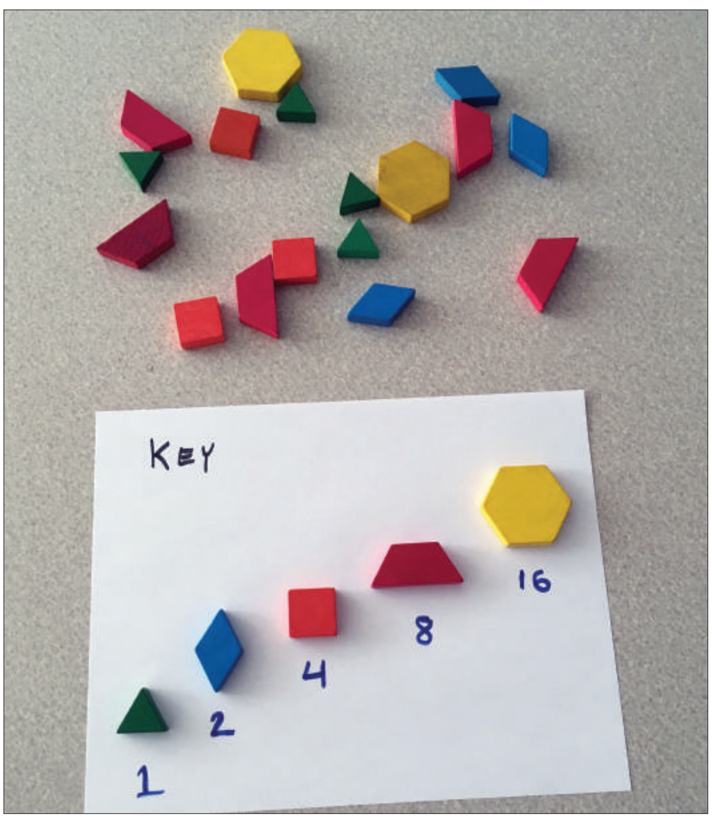
Figure 1. Data from the Hyperbinary Number investigation.

Question: What patterns do you see in the table of representations and in the values of the sequence $b(n)$?

Pattern 1: $b(n) = b(2n+1)$. There are a great number of patterns to find. We were very happy when one group of teachers immediately noticed repetition in the values of $b(n)$:

"We found that $b(1) = b(3)$, $b(2) = b(5)$, $b(3) = b(7)$, $b(4) = b(9)$, and $b(5) = b(11)$. The indices on the left side of each equality go up by 1 each time, and the indices on the right side of each pair go up by 2 each time. Thus, we conjecture that $b(n) = b(2n+1)$."

This is in fact the case, and we were able to prove it. Since $2n+1$ is odd, we need to use a 1 shape (Triangle) to represent it in hyperbinary. Removing this 1 shape leaves the number $2n$. Then, replacing each shape with the next lower shape divides the number by two, giving a representation of n.



Manipulatives, such as 3D-printed binary coins (top) and wooden tiles of different shapes (above), provide a hands-on way to explore and visualize the problem.

Pattern 2: $b(n) + b(n+1) = b(2n+2)$. This pattern is also important, but more subtle, and so we were ready to prompt teachers toward making this observation. Proving it is just a little more complicated than proving Pattern 1, but not too much more so. Suppose we start with a representation of $2n+2$. Since $2n+2$ is even, the hyperbinary representation must end in a 0 or a 2. If it ends in a 0, then chopping off this last 0 results in a representation of $n+1$. If it ends in a 2, then chopping off this 2 has the effect of subtracting 2 then dividing by 2. Hence it yields a representation of n.

Patterns 1 and 2 determine the sequence $b(n)$. So what is special about these two recurrence relation patterns? If we know the value of $b(0)$ and $b(1)$, then these two relations completely determine the value of any $b(n)$. In other words, the four declarations

$$b(0) = 1,\ b(1) = 1,\ b(2n+1) = b(n),$$
$$b(2n+2) = b(n+1)+b(n)$$

provide a complete definition of this sequence! Each odd term in the sequence is determined by a previous value, $b(2n+1) = b(n)$. And each even term is also determined by previous values of the sequence, $b(2n+2) = b(n+1)+b(n)$.

The Tree of Fractions

We then switched gears with our teachers, and put the hyperbinary sequence aside in order to investigate another structure with a binary nature which exhibits a lot of patterns. Secretly, we know that the teachers will soon see a connection with hyperbinary sequences!

The Calkin–Wilf binary branching tree of fractions, or Calkin–Wilf tree for short, is constructed with the following rules (see Figure 2 at right).

- The root node is at the top and is labeled 1/1.
- Every node has a left and right "child" node below it. If the node is labeled i/j, then its left child is labeled $i/(i+j)$ and its right child is labeled $(i+j)/j$.

Participants readily observe many patterns, for example: the numerators down the left side are always 1; the denominators down the right side are always 1; the reciprocal of any number appears in the "reflected" node on the other side of the tree; and the denominator of any node is the same as the numerator of the next node to the right. We enjoyed encouraging our participants to clarify these simple observations and try to explain them. Some are easy consequences of the definition of the tree; many will be useful in the next part of the investigation.

After some discussion, participants noticed that every reduced fraction appears once and only once in the tree. It is this property that we wish to focus on. First it is helpful to break this complicated observation down into its component parts:

(a) Every number in the tree is a reduced fraction,

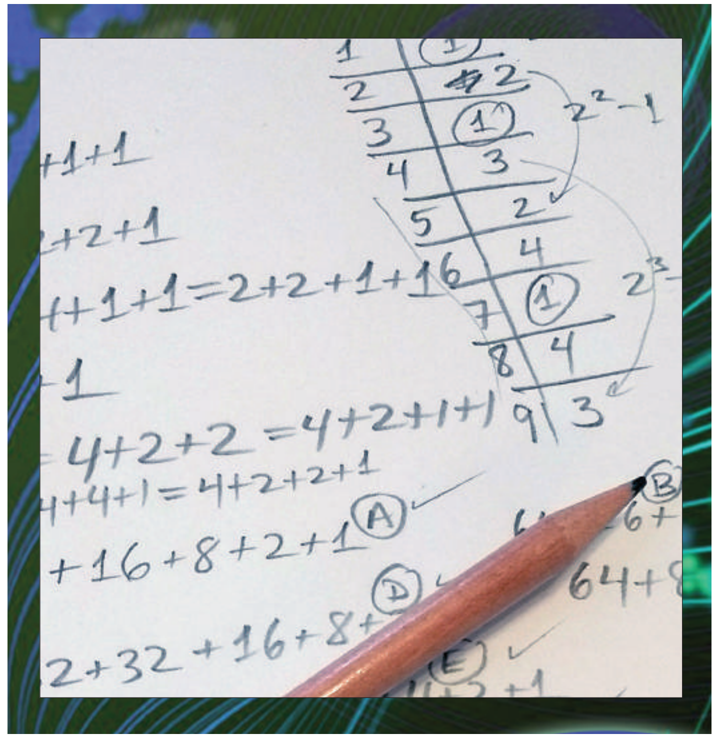
(b) Every positive rational number appears in the tree, and

(c) No number appears more than once in the tree.

Why is each of these three claims true? As a starting exercise, we asked the following:

Question: You were provided the rules to generate the tree "downward." What are the rules to generate the tree "upward?" That is, if a node is labeled r/s and the node is a left child, what is its parent labeled? And if a node is labeled r/s and it is a right child, what is its parent labeled?

The proofs of all three of these claims can be carried out by starting with a "least counterexample," which in this case means a counterexample that is as high in the tree as possible (having least level index). One then shows that it is possible to find a counterexample that is even higher in the tree, which gives a contradiction. It may be mentioned, emphasized, or formally stated, that this is equivalent to an induction-style argument on the level of the tree. For brief proofs of claims (a), (b), and (c), see our handout on proofs in the online version of this article.



Putting the two investigations together

Perhaps the most surprising observation that participants may make is that the hyperbinary numbers appear in the Calkin–Wilf tree. In fact, the sequence of denominators of each node, read left to right and then top to bottom, is exactly the hyperbinary sequence $b(n)$. Also, the sequence of numerators, excluding the top node, is the hyperbinary sequence

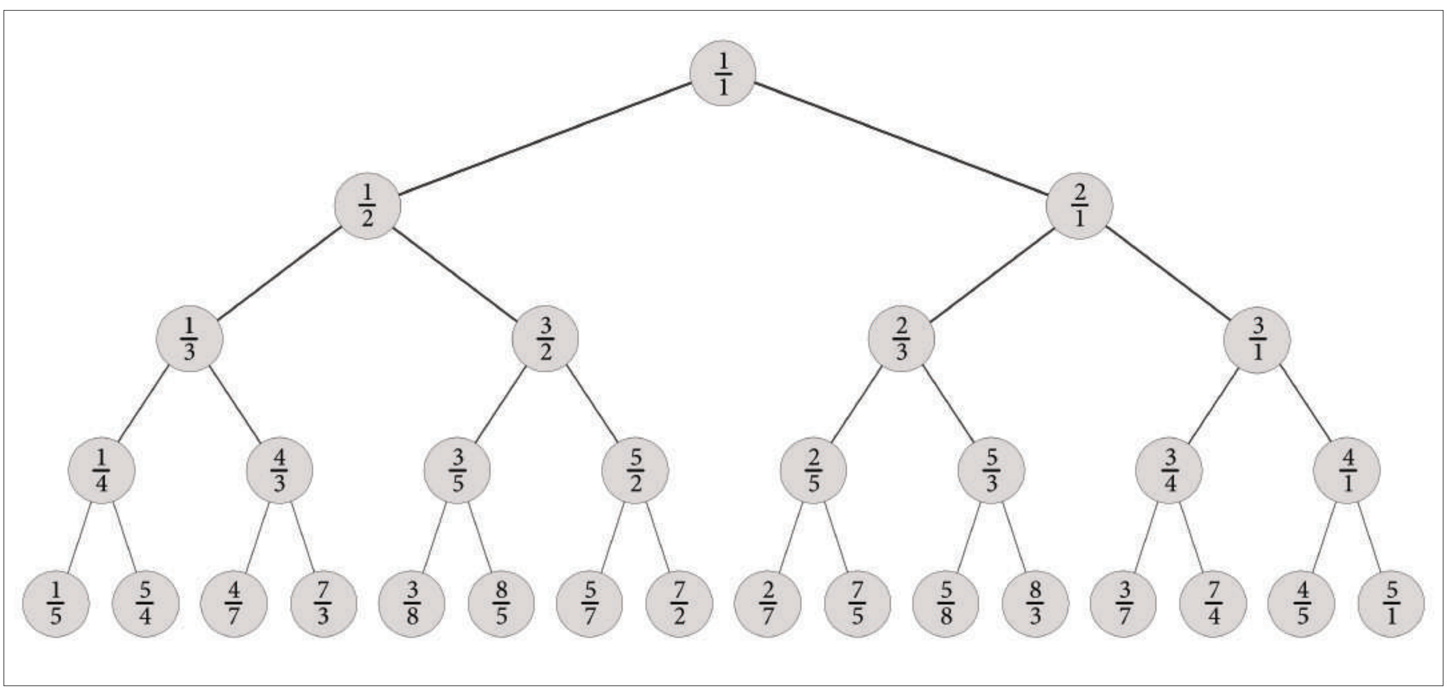


Figure 2. The Calkin–Wilf tree of fractions.

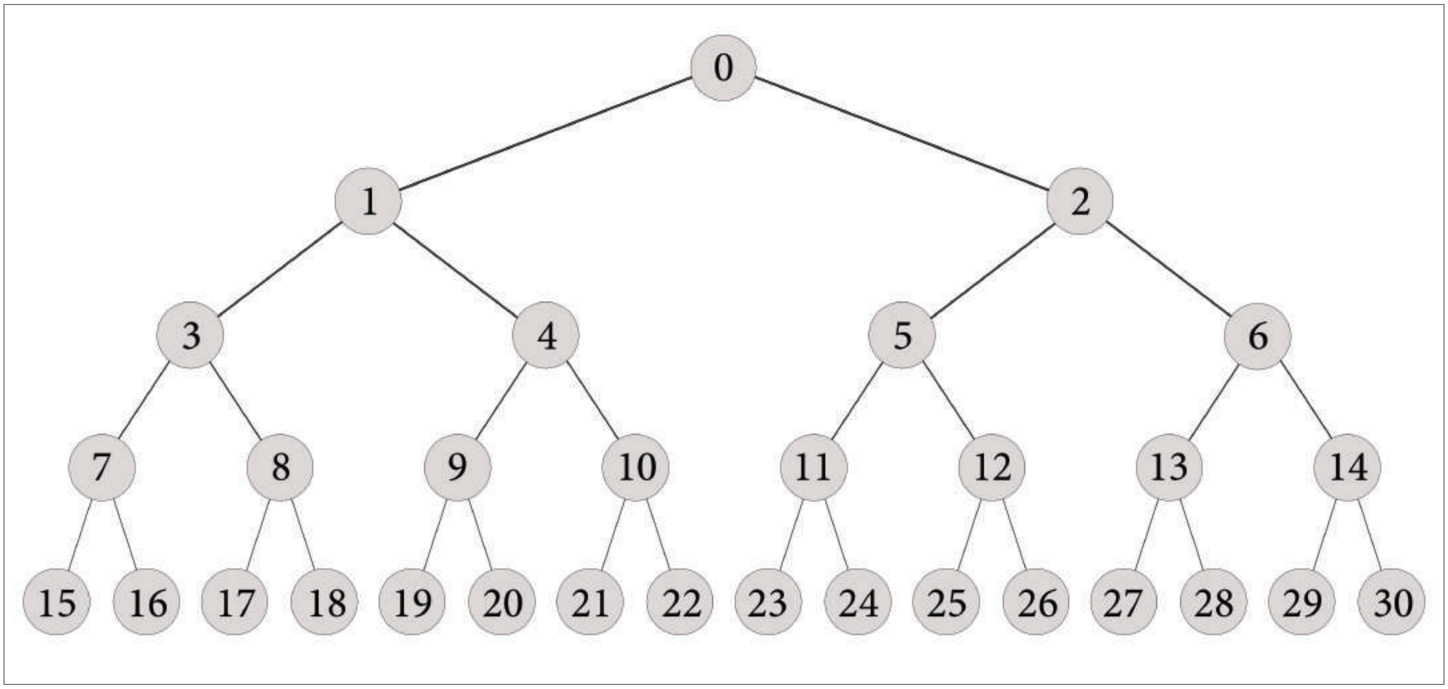


Figure 3. Numbering the nodes of the binary tree.

$b(n)$. The remainder of our sessions focused on uncovering, at least to some extent, why this is the case.

We again started by breaking the observation down into several components. In order to describe the components, we need to number the nodes of the tree in left-to-right, top-to-bottom order as in Figure 3 above. We may now phrase our claims as follows:

(d) The denominator of node n is the numerator of node $n+1$.

(e) The fraction label of node n has the form $f(n)/f(n+1)$ for some sequence $f(n)$.

(f) The sequence $f(n)$ is exactly the same as the sequence $b(n)$ explored earlier.

The proof of claim (d) is not difficult, but does involve three cases: Node n is a left child, node n is a right child not at the end of a row, or node n is at the end of a row. Claim (e) follows immediately from claim (d).

For claim (f), we can begin with the question, "What is the left child of node n?" The answer is always node $2n+1$, and this is a fun exercise on its own if there is time. This means the left child of the fraction labeled $f(n)/f(n+1)$ is the fraction labeled $f(2n+1)/f(2n+2)$. But then the definition of "left child" tells us that $f(2n+1) = f(n)$ and $f(2n+2) = f(n) + f(n+1)$. Do these statements look familiar? They should, because they are the patterns that we have said define the hyperbinary sequence $b(n)$!

We have arrived at the grand conclusion: If one writes out the sequence $b(n)$, which counts the number of hyperbinary representations of n, and then makes fractions out of the successive terms in the sequence, then one obtains an enumeration of all positive rational numbers, each in lowest terms, with no repeats! This completes Calkin and Wilf's elegant and explicit proof of Cantor's theorem, that there are exactly as many rational numbers as there are counting numbers.

Samuel Coskey (Boise State University and Boise Math Circle), Paul Ellis (Manhattanville College and Westchester Area Math Circle), and Japheth Wood (Bard College and Bard Math Circle) learned a great deal from each other in developing this activity and writing this article, which amalgamates their different experiences leading this activity in their Circles.

Resources

For links to this article's resources and more, visit us at **www.mathteacherscircle.org/newsletter**.

Listen, Share, Play

Lessons from Preschool for Problem Solving

by Peter Tingley

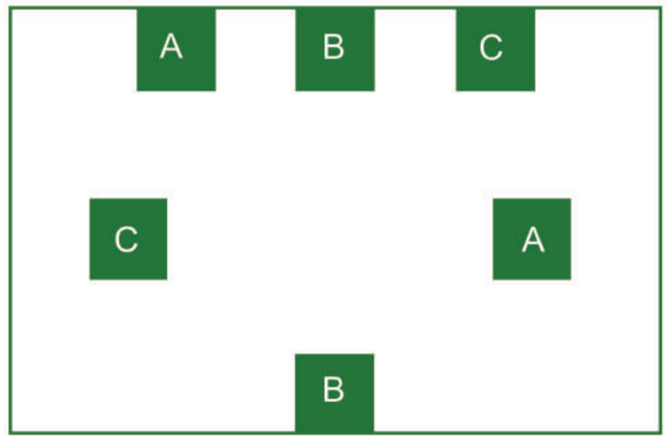
The main goal of our Math Teachers' Circle at Loyola University Chicago is to engage teachers in open-ended, interesting problem solving. In this article, I will talk about the problems I use to introduce what that means. I've used these a number of times with teachers, pre-service teachers, math majors, and even professors. But first I'm going to discuss a bit of philosophy.

I'm trying to engage teachers with good problems, to help them develop (and hopefully then teach) good problem-solving strategies and mindsets. But first, what is a good problem? For me it is a problem where when people first look at it, *they do not know what to do*. Solving it involves some exploration, which means doing things to see what will happen, not expecting them to lead directly to the answer. You'll see more what I mean when I introduce the problems.

Even more importantly though, what is a good problem-solving mindset? I've been led more and more to the language I hear from my 5-year-old. Her preschool has been doing a unit called I Can Problem Solve (ICPS). Well, in that context, it really means "Don't get in fights with the other 5-year-olds," but that is a kind of problem solving. Anyway, I'm talking about the following advice:

- ***Listen to the question.*** It has things to say, and they might be interesting. If you are always the one talking, you will miss out!
- ***Be willing to compromise with the problem.*** Maybe you won't solve it as stated. Maybe you should do an easier problem first. Maybe you'll have an idea for a similar problem that works out better, and you can learn from that. You'll have more fun if you can be flexible!
- ***Sometimes the problem wins, and that is OK!*** It isn't about who wins, its about how much fun you have together!
- ***You can keep playing even after the problem is solved!***

I usually talk about these things after first having people do the following problem, which I first learned about from Joshua Zucker:



Make paths connecting each pair of same-letter boxes so that none of the paths cross each other or leave the large box.

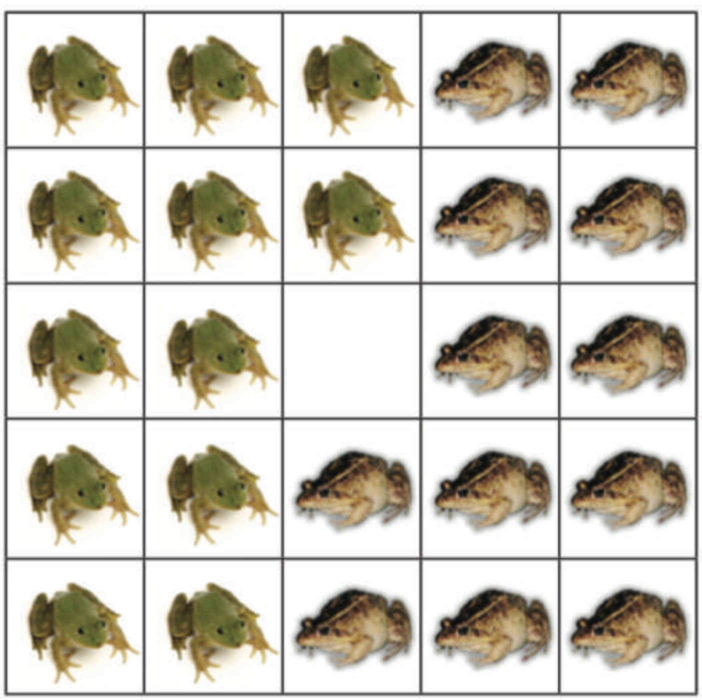
I ask them to do the problem individually, and allow only 3-5 minutes. My students once called this the "think outside the box but stay inside the box" question.

Tingley leads the Southwest Chicago MTC in a warm-up problem to start a conversation about strategies for problem solving.

I love this because none of the "formulaic" problem-solving techniques help at all. Draw a picture? Already done. Organize information? It already looks pretty organized.

But if you just start trying things and *listen to the problem* by thinking about what goes wrong, you will definitely solve it. If you *compromise* by simplifying somehow, say by first deleting the Cs, you will solve it (there are actually many interesting ways to simplify this question). In the end, if you've really understood things, you should be able to see what happens if, for example, I add a D box, or even an E and an F. That is, it is worthwhile to *keep playing,* even if you have a solution.

Then I do the frog and toad problem below. I got this from the summer 2013 MTCircular magazine, where (with slightly altered wording) it was the Problem Circle problem, also contributed by Joshua Zucker:



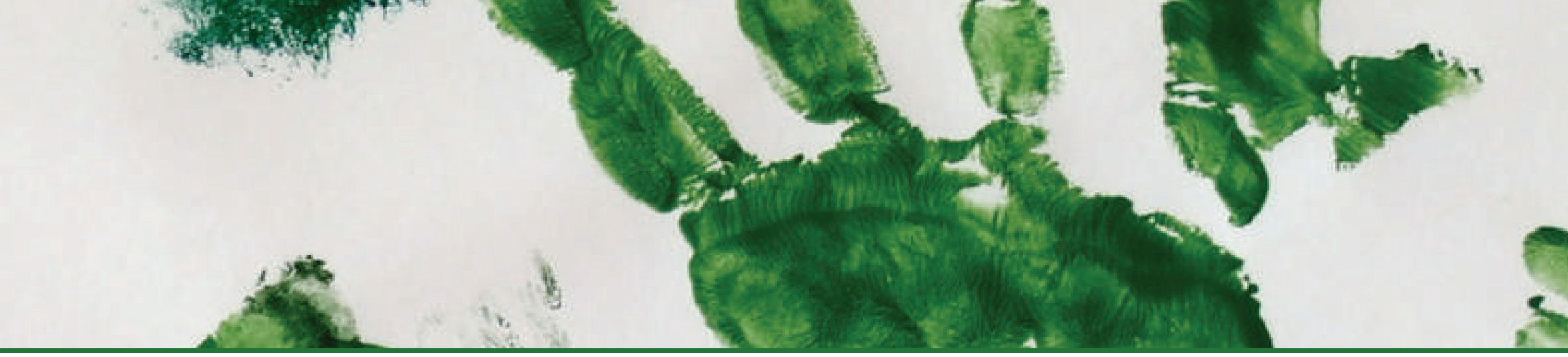
In this game, frogs can move right or down. Toads can move left or up. A frog or toad can slide into an adjacent empty space. They can also jump over exactly one of the other kind of animal to land in an empty space. They want to swap places, so that every place that now has a frog will have a toad, and vice versa. Can it be done? How?

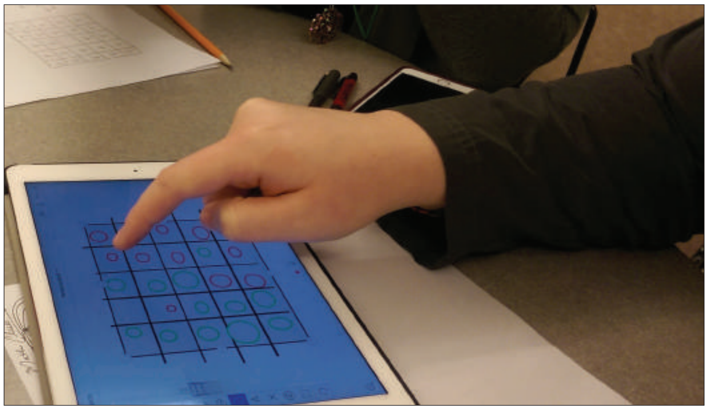
I have teachers do this in groups of 4, and plan on it taking an hour or so. Note that this is clearly not from the curriculum. That is a point I make: The solution in a sense doesn't matter. It is experiencing finding the solution that is worthwhile. It isn't immediately clear to anyone what to do (or even if it is possible), but you can start doing things. Very quickly, though, you see it is too complicated. You need to do a simpler problem! Most people choose to do a 3-by-3 version of the question, but this step takes longer then you might think.

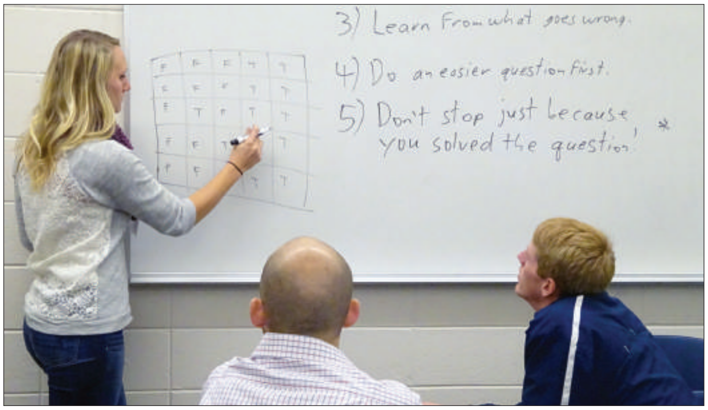
From there, there are three quite distinct ways to solve the problem, each of which gives insight into problem solving:

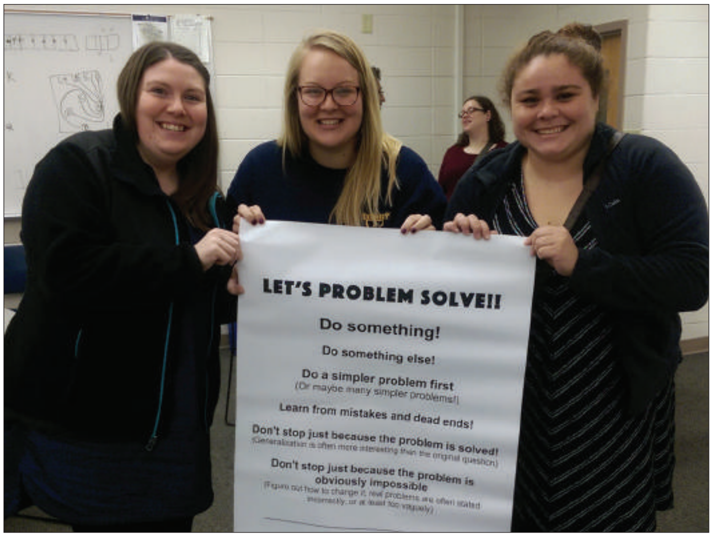
Method 1: Try and fail to do the 3-by-3 case a bunch of times. Keep a record of the stuck positions. Notice that most of these have three frogs at the bottom or three toads at the top, which is bad (not quite all have this property). Conclude: Moving up or down is dangerous...which you should have known! The frogs and toads mostly need to go left-right, not up-down! Systematically go left-right as much as possible. Now you'll solve it.

Method 2: Solve the 3-by-3 case by trial and error, then organize your solution so you can describe it nicely in words, at which point it should generalize. This can be problematic, because there are solutions that don't generalize well. But that just gives a chance to talk about the value in looking for better solutions once a problem is solved. It also gives a way to understand what a better solution is: It is one that is easier to explain and generalize.









Southwest Chicago MTC members make good use of problem-solving strategies in the Frogs and Toads problem.

Method 3: Be more creative about generalizing the problem! For example, it makes sense for a 3-by-5 rectangle, or 1-by-3! A 1-by-3 rectangle is easy. A 1-by-5 rectangle is a bit harder, but you'll get it. And now you can notice, in the original 5-by-5 question, you can start by flipping the middle row using the 1-by-5 case! Then move one animal up or down and flip another row. Now you'll get it...

Method 3 is my favorite, and I push some people towards it a bit. But I mostly try to let people go their own course. Because actually my favorite thing is that there are so many nice solutions, and that you can have great discussions about them.

There are extensions for people who finish early (not many do, in my experience). You can ask how many moves it takes (I know an answer for my solution, but not a proof that all solutions will have the same number of steps, although I think they do). You can do different rectangles. You can even think about what to do if one of the lengths is even (so the initial hole has to be a bit off-center). I don't know the full answer to that either. So, there is plenty left to explore!

Peter Tingley, a co-founder of the Chicago MTC, is an Associate Professor of Mathematics at Loyola University Chicago.

Tiling With Pentagons

Exploring Newly Discovered Math

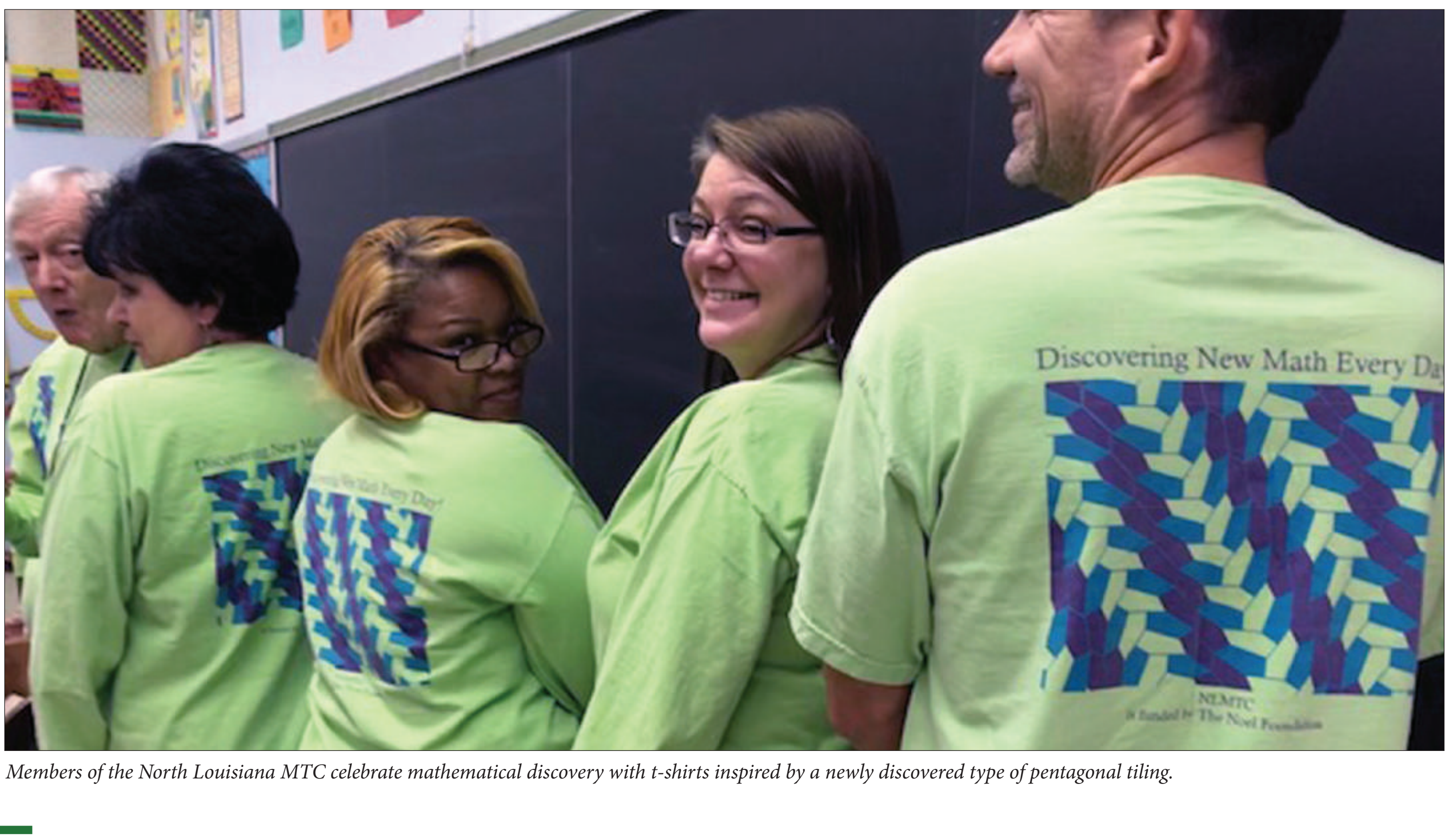
by Judith Covington

On August 14, 2015, Casey Mann, Jennifer McLoud, and David Von Derau of the University of Washington Bothell announced that they discovered a new irregular pentagon that would tile the plane. Prior to this announcement there were only fourteen known types of pentagons that tile the plane. In 1918, Karl Reinhardt discovered the first five, and in 1968, Richard Kerschner discovered three more. In 1975 Richard E. James III found a ninth. Marjorie Rice, an amateur mathematician, discovered four new types in 1976 and 1977. In 1985, Rolf Stein discovered the fourteenth. Then, in 2015, the publication of the fifteenth pentagon appeared. (Interestingly, in 2017, Michaël Rao proved that there were no more than these fifteen.)

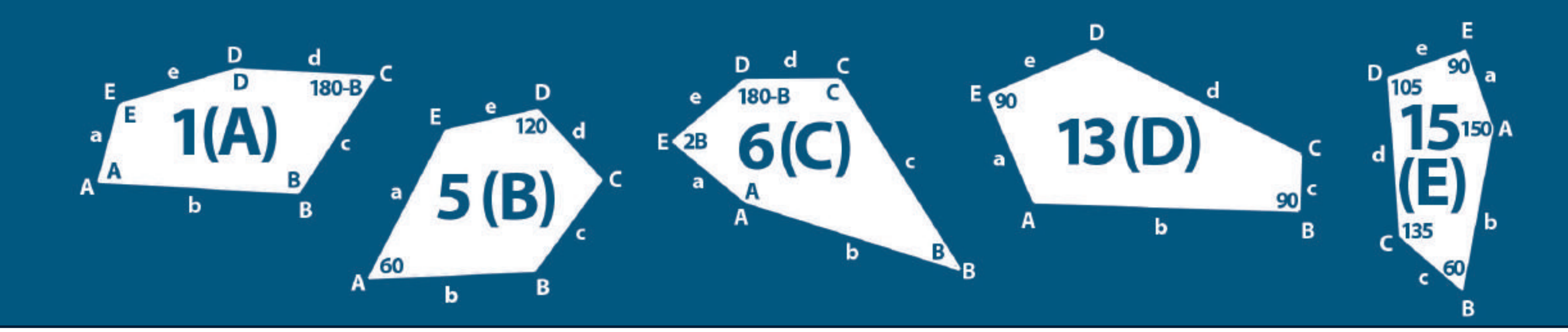
"Tiling with Pentagons" became one of the best sessions of Fall 2015 for the North Louisiana Math Teachers' Circle in Shreveport, Louisiana. The participants loved working with newly discovered mathematics that they could use in their own classroom.

To start the session, we did a quick review of convex polygons and their properties. In particular, the participants reviewed the names of certain polygons and worked to find the measure of each angle of a regular polygon with sides from three to twelve. The teachers recalled that a regular polygon has sides of the same length and angles of the same measure. Starting with the fact that the angles of a triangle have a sum of 180 degrees, teachers were encouraged to find a formula that would work for any convex polygon.

After mastering this information about convex polygons, we moved on to regular tessellations. A regular tessellation uses copies of the same regular polygon to cover a flat surface (plane) with no holes and no overlap. Furthermore, the number of polygons meeting at each vertex should be the same for all vertices. Teachers used the term "covering a point" to indicate the filling of all space surrounding a vertex. This meant that the sum of the angles meeting at one



Members of the North Louisiana MTC celebrate mathematical discovery with t-shirts inspired by a newly discovered type of pentagonal tiling.



vertex must be 360 degrees. The participants quickly discovered that the triangle, square and hexagon are the only regular convex polygons that will tessellate. When asked for a proof of this fact, the teachers shared that these are the only regular polygons whose individual angle measures are factors of 360 degrees.

The discussion then turned to other ways to tessellate. Possibilities discussed were: allowing the use of more than one shape; allowing the number of polygons meeting at a vertex to be different; or using non-regular polygons. With a bit of guidance, the participants focused on whether there are any non-regular convex polygons that always tessellate. The discovery was that any triangle or any convex quadrilateral will tessellate. Arranging six copies of a triangle so that each angle of the triangle appears twice at each vertex will cover the plane. The sum of the angles of a triangle is 180 degrees and if two copies of each angle meet at a vertex, the sum will be 360 degrees. Similarly, any convex quadrilateral will also tile the plane since the sum of the angles in a convex quadrilateral is 360 degrees. Arranging the quadrilaterals so that each angle meets at one vertex will cover the plane.

Next, we discussed pentagons. The participants knew that not all pentagons would tile the plane, since the regular pentagon does not tile the plane. However, they wondered if there were any convex pentagons that would tile the plane.

At this time, the teachers split into five groups. Each group received a few card stock sheets of a unique pentagon and were asked whether that pentagon would tessellate. We called the different pentagons A, B, C, D, and E. The teachers cut out multiple copies of their pentagon and arranged them on tables. Some of the pentagons proved easy to tessellate. In other cases, the participants reached the decision that the shape would not tessellate.

As the teachers struggled to get some of the shapes to tessellate, we gave some additional information about the sizes of the angles, in the hope that this would allow them to see arrangements of angles that would add to 360 degrees. Some of the groups found the additional information helpful and were then able to create a tessellation, but some groups still struggled with the more difficult shapes.

Shape One (A)	B + C = 180°, A + D + E = 360°
Shape Five (B)	a = b, d = e, A = 60°, D=120°
Shape Six (C)	a = d = e, b = c, B + D = 180°, 2B = E
Shape Thirteen (D)	d = 2a = 2e, B = E = 90°, 2A + D = 360°
Shape Fifteen (E)	a = c = e, b = 2a, A = 150°, B = 60°, C=135°, D = 105°, E = 90°

After each group had time to explore and share their experiences, they learned that all of the pentagons provided would cover the plane. This lead to a discussion of the history of tessellating with convex pentagons. At this point, the teachers were informed that Shape E had been discovered only a few months prior to our meeting and was the fifteenth type of polygon that would tessellate. This led to an exciting discussion of the methods used to discover this new type. To end the meeting, we shared links to recent press items.

A few months later, when discussing artwork for a new t-shirt for the group, the teachers wanted to put the fifteenth pentagon tiling on the back of our shirts with the words, "Discovering New Math Every Day!"

Working with newly discovered mathematics was a thrilling experience for all. There were no textbooks or daunting formulas, just observations, patterns, discussions, and lots of inquiry.

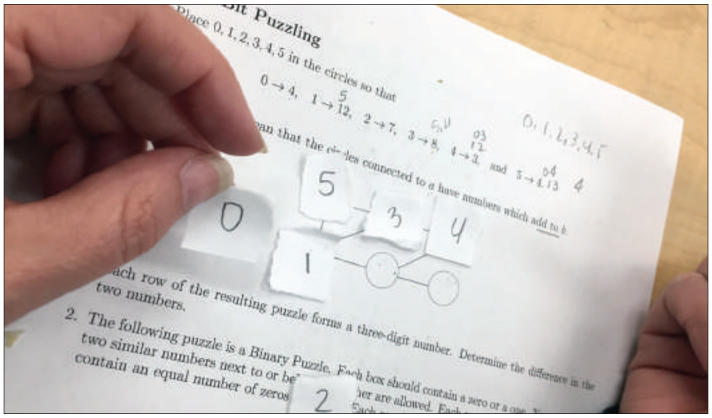
For links to resources, references, and news items related to this session and more, please visit **http://www.mathteacherscircle.org/newsletter**.

Judith Covington, a co-founder of the North Louisiana MTC, is a Professor of Mathematics at Louisiana State University Shreveport.

Locked Out

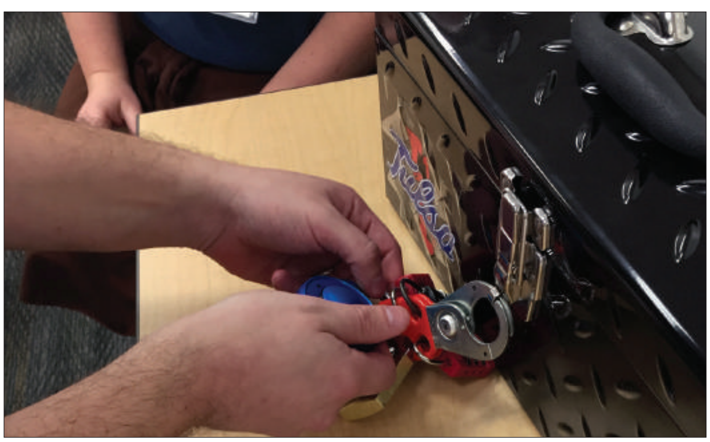
A Breakout Box Session for Your Circle

by Kimberly D. Adams and Donna S. Farrior



Sample problems from the Breakout Box session.


Tulsa MTC members work together to solve a puzzle.


A correct solution produces the combination to a lock.

Math Wrangles and Math Auctions are well-known Math Circle activities in which teams compete for the best solutions of a set of engaging and challenging problems. The Tulsa MTC has used these formats to great success in previous meetings. Outside the math circle arena, Escape Rooms and "Bomb Disposal" activities are growing in popularity as a form of team building and entertainment.

This fall, we blended the two ideas to create a co-operative math activity where the challenge is to solve math problems whose solutions generate combinations to open a locked box. The math problems can be selected to fit any audience, and the activity appeals to problem solvers of all ages.

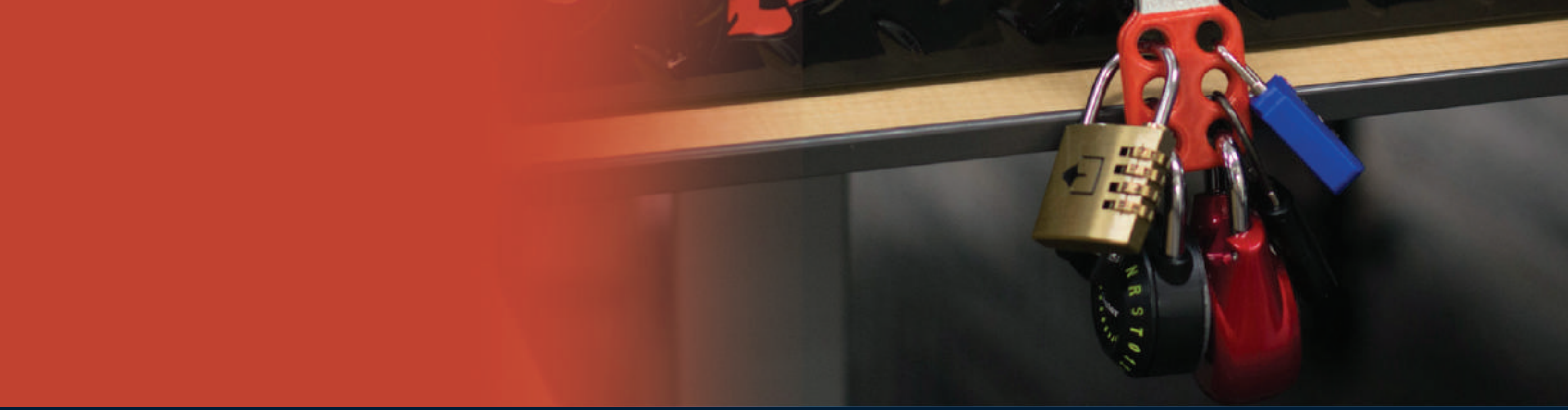
Materials

- Lockable box
- A lockout hasp with holes for multiple locks (shown in photo on header, above right)
- A variety of combination locks
- Handout with problems
- Handout with lock instructions

We used six locks per box: several three- and four-digit combination locks, a directional lock, and a word lock. Breakout Kits may be purchased directly from BreakoutEDU (https://www.breakoutedu.com), but we built our own kits at around $40 per kit. If you can afford it, we recommend making one kit for every six to eight participants.

A word to the wise: Keep up with the combinations, as the locks are the most expensive part of the kit. (Remember losing your locker combination in junior high?) We made a one-page master set of instructions for opening all the locks in our kits.

So, what's in the box? Maybe a letter to the participants, or some small prizes. Maybe another locked box with one additional puzzle. It doesn't really matter!



The anticipation of solving puzzles, the fun of working together, and the reward of opening the box can outweigh any physical prizes.

Preparing for the Session

The beauty of the Breakout Box session is that the problems can be tailored to your group, and can change every time you do it. For a fun teacher session, you can choose to do one challenging problem per lock, or a few easier problems. Here are our suggestions for picking problems:

- Include problems from a variety of topics: geometry, counting, number theory, logic, etc.
- Use problems with multiple entry points. More entry points = more opportunities for teamwork.
- Avoid problems where the solution can be guessed from a small finite set, such as a Knights and Liars problem: "Who was lying? Tom or Joe?"
- Once you have selected your problems, consider eliminating two or three so as to not overwhelm the participants.

Suitable problems may be found in student math contests as well as magazines and puzzle columns. A small list of suggested problems and resources can be found in the online version of this article.

When designing your session, keep in mind that you can tack on instructions for creating a combination from the results. For example, say that four problems have solutions 27, 15, 45 and 120. To create a combination for a directional lock, you might ask participants to convert each solution to its mod 4 equivalent (3, 3, 1, 0) and then assign a direction to each result (0=up, 1=down, 2=left, 3=right) to produce the combination right, right, down, up.

We cannot stress it enough: triple-check your materials and handouts before the session begins. It is very frustrating for participants if we have made an error in the calculation to a combination lock!

The Session

As participants arrive, place them in groups of six to eight. Late arrivals can form a new team, or join an existing team. Give each group a set of the lock instructions and the problem list, and tell them the time limit. Then start the clock running!

Participants will become aware that brute force methods are not an efficient use of their time. Rather, these problems require creativity and productive work. You may be called upon to verify the correctness of a solution on the spot if a lock won't open.

When the first team opens its first lock, the excitement in the room starts to build. Depending on how competitive your group is, you can relax the rules towards the end of the session, and allow teams to help each other. During our session, one team had only one lock remaining at the end, and another team helped them find the combination.

Takeaways

One of the biggest challenges with this activity is time management. A normal Math Teachers' Circle session can be shortened or lengthened on the fly. However, the timing of a Breakout Session is set the moment you distribute the materials.

At our first Breakout Session, no team actually opened their box in the allotted 90 minutes without help. Surprisingly, this was not an issue. The excitement of opening each lock kept the groups energized and productively working throughout the session. Opening a single lock is a big victory!

Kimberly Adams is an Instructor and Donna Farrior a retired Instructor in the Mathematics department of The University of Tulsa. Both authors are co-directors of the Tulsa Girls' Math Circle, and are members and occasional facilitators at the Tulsa Math Teachers' Circle.

Bolognese Wins Statewide Teaching Award


Bolognese

Chris Bolognese, co-founder of the Columbus Math Circle and chair of the Columbus Academy math department in Columbus, Ohio, was honored with this year's Buck Martin Award for Exemplary Mathematics Teaching by the Ohio Council of Teachers of Mathematics (OCTM). This statewide award is given to one secondary teacher in the state of Ohio each year. To be considered for this award, the teacher must have previously been chosen as the outstanding secondary mathematics teacher in one of the eight districts in Ohio. In response to the award, Bolognese said, "I'll never stop learning as a teacher and will always support my students and colleagues to take on new challenges and risks." To watch an NBC4 segment featuring Bolognese and how he fosters problem-solving skills in the classroom, please visit https://tinyurl.com/columbusmath.

Finkel, Jones Present at International Conferences

Dan Finkel, a leader of the Seattle MTC and director of the mathematics education consulting business Math for Love in Seattle, Wash., went on a five-week Australian speaking tour, where he gave keynote addresses at the MAV and MAWA conferences (Mathematics Associations of Victoria and Western Australia, respectively).

Troy Jones, a leader of the Utah MTC and teacher at Westlake High School in Saratoga Springs, Utah, gave a presentation in Spanish at the Congreso Internacional de Cabri at the University of Medellín, Colombia, about the points of concurrency in a tetrahedron. Jones will travel to Quito, Ecuador, in February to help with a workshop for secondary math teachers.


Finkel


Jones

Shubin Honored with Mary P. Dolciani Award


Shubin

Tatiana Shubin, co-founder of the Math Teachers' Circle Network and a professor at San Jose State University, was honored with the 2017 Mary P. Dolciani Award at the Mathematical Association of America MathFest in Chicago in July. The Dolciani Award annually recognizes a pure or applied mathematician who is making a distinguished contribution to the mathematical education of K-16 students. Shubin has dedicated her life to bringing Math Circles to new communities, particularly indigenous populations. Her work developing the Navajo Math Circles and the Math Teachers' Circle Network are just two of her distinguished contributions to mathematics education in the U.S.

Engle, Danforth-Clayson Win Rosenthal Prize

Matt Engle, member of the AIM MTC and a teacher at Monterey Bay Academy in La Selva Beach, Calif., is the winner of the 2017 Rosenthal Prize for Innovation and Inspiration in Math Teaching (see application announcement below). Engle was awarded a $25,000 cash prize for his lesson "Bringing Similarity Into Light: Experiencing Similarity and Dilations Using Shadows." In Engle's lesson, students examine the shadows of shapes to explore concepts such as ratio, dilation, and proportionality in triangles.

Engle

Danforth-Clayson

The 2017 second-place winner of the Rosenthal Prize is Heather Danforth-Clayson, also an AIM MTC member and a teacher at Helios School in Sunnyvale, Calif. Danforth-Clayson developed her lesson, "Derangements and Random Rearrangements: An Exploration of Probability," during an AIM MTC teacher leaders' workshop in 2016. The lesson explores rearrangements of number sets by conducting hands-on experiments in probability.

OPPORTUNITIES

Applications Open for 2018 Rosenthal Prize

MOMATH NATIONAL MUSEUM OF MATHEMATICS

The Rosenthal Prize for Innovation and Inspiration in Math Teaching, a $25,000 prize administered by the National Museum of Mathematics in New York City (www.momath.org), celebrates innovation and inspiration in the upper elementary and middle school math classroom by rewarding a teacher who creates an exceptional, hands-on, math activity. The application process opens in January 2018 and preliminary applications are due in May 2018. Find out more at momath.org/rosenthal-prize or by emailing rosenthalprize@momath.org.

Give Students a Competitive Edge with the MAA American Mathematics Competitions

MAA AMC American Mathematics Competitions

Problem-solving skills are at the core of mathematics. Join the MAA American Mathematics Competitions (AMC) for the opportunity to build creative problem-solving skills in your math students. The AMC is a great way for students to hone their analytical thinking and problem-solving skills through classroom resources and friendly competition. The middle school competition takes place in November and both levels of the high school competitions are held in February. Visit amcreg.maa.org to learn more and register for AMC.

Know of another opportunity of interest to the MTC community? Contact us at circles@aimath.org.

Dispatches from the Circles

Local Updates from Across the Country

California •

We are developing **Bay Area Teachers and Mathematicians (BATMath)**, as an alliance of MTCs hosted in counties that touch the San Francisco Bay. BATMath is now a network of 10 MTCs, and growing! This summer, AIM held an immersion workshop for 35 newcomers to BATMath. Meanwhile, six teacher leaders — all veteran MTC members — spent the week writing middle school lesson plans based on topics presented at the workshop. You can find these lesson plans online at http://batmath.org.

– Contributed by Hana Silverstein

Middle school teacher and **San Diego MTC** member David Honda displayed a work of mathematical art at the 2018 Joint Mathematics Meetings. Honda's "Dodecahedral 11-Hole Torus" was inspired by the challenge of creating an origami structure with both concave and convex surfaces.

– Contributed by Yana Mohanty

Kentucky •

The **West Kentucky MTC**, recently founded at Murray State University, received a generous $4,000 grant from the Peter H. and E. Lucille Gaass Kuyper Foundation. The Foundation is closely tied to Pella Corporation, one of the largest employers in Murray, KY. We did "Exploding Dots" at our first MTC recruiting workshop in November at a local middle school, and it was a resounding success.

– Contributed by Craig Collins and Beth Donovan

Mississippi •

The **Mississippi Delta MTC** will host a Celebration of Mind event on February 24, 2018. We also hosted Saturday MTC sessions in August, November, and January.

– Contributed by Liza Cope

New Mexico •

In July 2017, 29 teachers attended the **Northern New Mexico MTC** summer institute, led by James Taylor (Santa Fe MTC), Amanda Serenevy (Navajo Math Circles), and Bob Klein (SouthEast Ohio MTC). The institute was made possible by the Northern New Mexico College STEM Mentor Collective, Española TechHub, Math Circles Collaborative of New Mexico, NM Supercomputing Challenge, and Los Alamos National Laboratory (LANL) Foundation.

Math Circles of Northern New Mexico partnered with Española Valley High School (EVHS), Pojoaque High School, and Carlos Vigil Middle School (CVMS) to host math circle activities in the schools. CVMS invited us to take over 7th grade math lessons two Fridays a month to regularly reach all 250 students at the school! James Taylor and Melany Rodriguez (Española TechHub) facilitate the circles. Española TechHub received a $6,132 grant from LANL Foundation to further math circle activities in Northern New Mexico schools.

– Contributed by Josephine Kilde

Pennsylvania •

The first MTC in central Pennsylvania, **Math Teachers at the Center of Pennsylvania (MTCPenn)**, held a successful three-day workshop at Juniata College with support from national MTC mentor Bob Klein. The seed that we planted has started to grow and we hope that it will flourish and persist for a long time.

– Contributed by Henry Escuadro

Utah •

In Fall 2017, the Utah MTC held sessions exploring perspective drawing and string art. In January 2018, we will explore the mathematics behind kicking a conversion in rugby.

– Contributed by Troy Jones

Global Math Week

“Exploding Dots” From Coast to Coast

During the first annual Global Math Week, beginning October 10, 2017, a million and a half students experienced Exploding Dots, a refreshing and powerful mathematical approach to arithmetic and algebra developed by founding team member James Tanton and regularly described by students and teachers alike as “mind-4Wblowing.” Here is how members of the MTC community across the country celebrated Global Math Week.

Clockwise from top left: Dan Finkel (Seattle MTC) shared exploding dots and reflective geometry with Hillcrest Elementary students in Lake Stevens, Wash.; Chris Bolognese (Columbus MTC) did the math salute and introduced exploding dots at a Columbus Academy assembly in Columbus, Ohio; Global Math Project founder James Tanton celebrated the start of Global Math Week in Phoenix, Ariz.; Hilary Kreisberg presented exploding dots at the Boston MTC in Cambridge, Mass.; Brandy Wiegers (national MTC mentor) exploded dots with Central Washington University students in Ellensburg, Wash.

American Institute of Mathematics
600 E. Brokaw Road
San Jose, CA 95112

NONPROFIT ORG
U.S. POSTAGE PAID
AUSTIN, TX
PERMIT NO. 882

MTC Network Leadership Team

- Estelle Basor, *AIM*
- Brian Conrey, *AIM*
- Tom Davis, *San Jose Math Circle*
- Brianna Donaldson, *AIM*
- David Farmer, *AIM*
- Mary Fay-Zenk, *Consultant*
- Tatiana Shubin, *San Jose State University*
- James Tanton, *MAA*
- Paul Zeitz, *University of San Francisco*
- Joshua Zucker, *AIM and Julia Robinson Mathematics Festival*

MTC Network Sponsors

NSF
AMS American Mathematical Society www.ams.org
MAA
MfA Math for America
EAF
National Security Agency United States of America

desJardins/Blachman Fund

To subscribe to MTCircular or to explore resources featured in this issue, please visit us online at www.mathteacherscircle.org/newsletter.

Comments and suggestions are always welcome at circles@aimath.org.

MTC Network Advisory Board

- Kristen Chandler, *MATHCOUNTS*
- Wade Ellis, *West Valley College (Emer.)*
- John Ewing, *Math for America*
- Richard Grassl, *University of Northern Colorado (Emer.)*
- Edward M. Landesman, *University of California Santa Cruz (Emer.)*
- W. J. “Jim” Lewis, *University of Nebraska-Lincoln*
- Jane Porath, *East Middle School, Traverse City, Mich.*
- Richard Rusczyk, *Art of Problem Solving*
- Alan Schoenfeld, *University of California Berkeley*
- Denise Spangler, *University of Georgia*
- Peter Trapa, *University of Utah and MSRI*
- Uri Treisman, *University of Texas at Austin*

MTCircular

- Brianna Donaldson, Editor
- Hana Silverstein, Assistant Editor
- Jessa Barniol, Art Director
- Editorial Contributors: Kimberly D. Adams, Samuel Coskey, Judith Covington, Paul Ellis, Donna S. Farrior, Peter Tingley, Japheth Wood

MTCircular